Grids and Squared Paper in Renaissance Architecture —

Fabio Colonnese

Fig.1 Anon., Manuscript album of designs for lace and embroidery, German, *c.*1596. The figures were drawn by the owner on a woodcut printed grid. Overall dimensions 20 x 14cm. Acc. no. 53.566.7 (fols 6v–7r), Metropolitan Museum of Art, New York. Gift of Mary M. Greenwood, in memory of Eliza Rudd Greenwood, 1953.

There was a time when a piece of squared paper was the architect's favourite surface on which to develop their project. Thus, by the mid-18th century, we find Bernardo Vittone's *Istruzioni Elementari*[1] including 'a great number of plates in which the grid was used for the determination of plans of buildings and gardens, for the composition of elevations, and as the basis for tracing abstract geometrical figures or emblems' such as columns, walls, openings and whole façades.[2] A decade later, the Neapolitan Nicola Carletti endorsed the grid as a multi-purpose system to encourage 'the universal practice of arts'.[3] For his part, Thomas Jefferson explored drawing on a squared paper that was produced in Paris for silk-weavers and presumably favoured the use of metric – as opposed to imperial – measurement in some of his projects.[4]

In 1794, while Jean-Nicolas-Louis Durand was presenting the grid as a way of combining the architect's exaltation of invention with the engineer's attention to economic and constructional issues,[5] the physician Dr Buxton began selling rectangular gridded paper sheets in London.[6] 'Such charts may be obtained, neatly engraved,' wrote the astronomer Frederick William Herschel years later, 'and are so very useful for a great variety of purposes, that every person engaged in astronomical computations, or indeed, in physico-mathematical inquiries of any description, will find his account in keeping a stock of them always at hand.'[7]

During the 19th century, parallel to the spread of transparent paper so reviled by Gottfried Semper,[8] the production of squared paper increased – a reflection of the efficiency of its use in industrial processes. Graph or co-ordinate paper featuring a tighter and differentiated grid was created to encourage the drawing of mathematical functions. This was in part a consequence of the so-called 'Perry Movement', named after the president of the Institute of Electrical Engineers in Great Britain in the 1910s, John Perry, who promoted a reform in mathematics education that soon extended to the United States. In 1900, he explicitly wrote that 'the practical engineer needs to use squared paper',[9] emphasising its facility for the drawing of graphs and diagrams. But this paper was soon adopted by architects more generally to aid the metric control of the architectural drawing and papers with special geometric grids were also designed for supporting the construction of axonometric and perspective views.

While squared paper is still currently used in schools, most architects have dismissed it. The massive expansion of digital drawing has been reshaping their design processes, the organisation of their offices, their media, and their construction procedures. New digital tools, initially conceived as reproducing traditional ones,[10] have also replaced grids and squares or simply made them 'invisible',[11] and have subtly reshaped their use in architectural form. 'What we have to write contributes to the formulation of our thoughts', wrote Friedrich Nietzsche about his new typewriter.[12] In addition, computerisation has contributed to a retrospective reading of previous design processes, casting a fresh light on prior practices. From this perspective it is, for example, possible to discern forerunners of pixels in documents such as that illustrated in Fig.1 – a curious late 16th-century book, a prototype of the squared notebook, whose pages present a large, dense and empty xylographic grid to facilitate the design of patterns for tapestry and fabric.

Exloring the origins of the use of squared paper in architecture, this article will focus on the Renaissance as a key moment in the rise of the grid as a proportional system that is posed against – but is also complementary to – both medieval and Vitruvian compositional procedures. After tracing the several uses of the grid back to early 15th-century applications, I will discuss – based on an analysis of materials held in the Uffizi – a group of architectural designs on squared paper from Bramante's circle as a manifestation of this trend. A number of applications of the grid in the practice of 16th-century architects, occasionally connected to the motif of the labyrinth, are then described to demonstrate the developing use of squared paper in the design of military buildings, gardens and 'extra-large' projects, which thereby gradually lead to a flexible, universal system of design that will, in turn, promote the presence of the grid as an organising substrate.

Renaissance grids

The use of square grids is generally linked to Greek civilisation, mathematics, the city planning of Hippodamus of Miletus, and the square module of Hellenistic temples, such as that of Athena Polias on the hill of Priene. There is evidence that the Egyptians and Romans used some in the design of sculptural elements, to scale and control engravings and excavations[13] – and also to reproduce them – while the astonishing stone-carved map *Yu Ji Tu*, the *Map of the Tracks of Yu Gong* (1137), demonstrates its use in 12th-century Chinese cartography (and, indeed, earlier).

To understand the early diffusion of squared paper in the Renaissance, it is first important to focus on the many uses of the grid in artistic practices during the period when a design system based on drawing on paper developed.[14] While artisans had learned to square paper before tracing designs to be reproduced on fabrics and tapestries,[15] the development of the theory of perspective produced a primary shared operative framework in which artists and architects experimented with the grid. Thanks to Leon Battista Alberti, painters learnt how to construct drawings starting from a square grid placed in perspective on the ground that became embodied in elegant, coloured, chequered floors in paintings. Piero della Francesca also developed grid-based procedures to bypass the problem of using distant vanishing points on the wall and to determine the foreshortening of bodies in depth in a mathematical way.[16]

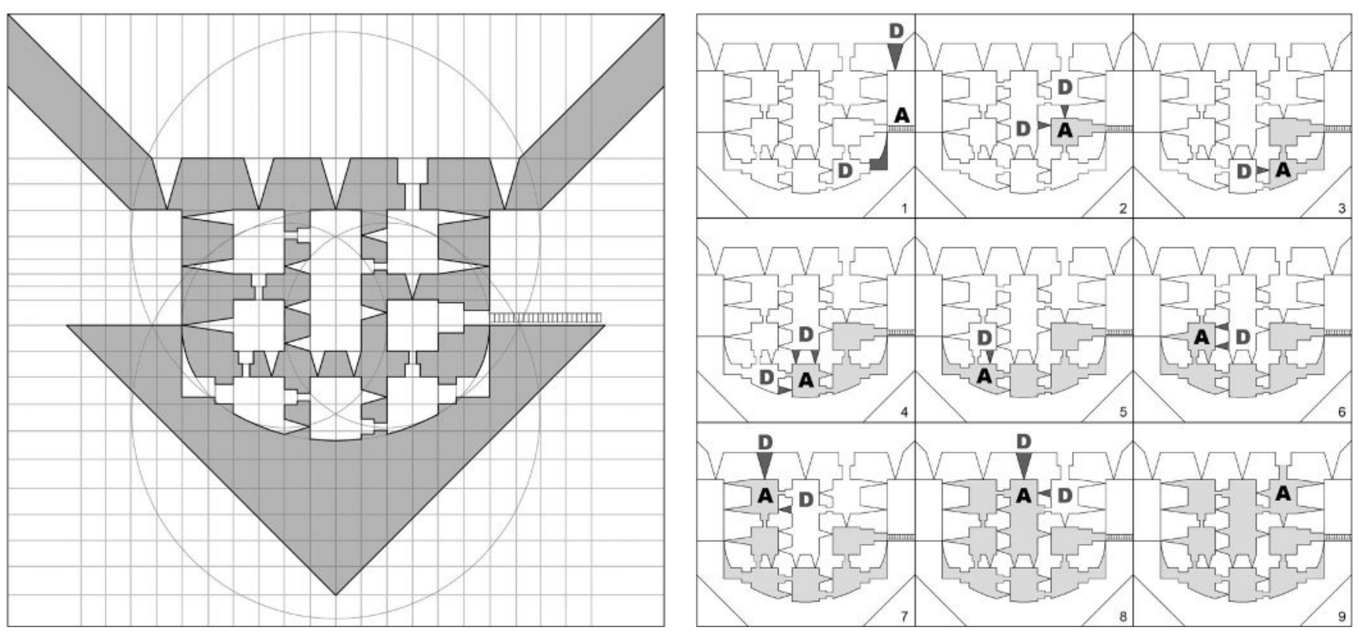
Fig.2 Raffaello Sanzio, *The Liberation of St Peter*, Vatican Palaces, 1513–14. Detail of the prison. Raphael, Public domain, via Wikimedia Commons, https://commons.wikimedia.org/wiki/File:Raphael_-_Deliverance_of_Saint_Peter.jpg [accessed 03.01.24].

Fig.3 Albrecht Dürer, Draughtsman making a perspective drawing of a reclining woman, *c.*1600 (originally from Dürer's *Unterweysung des Messung*, 1525). Woodcut, 7.7 x 21.4cm. Acc. no. 17.37.314, Metropolitan Museum of Art, New York. Gift of Henry Walters, 1917.

Raphael's *The Liberation of St Peter* (Fig.2) in the Hall of Heliodorus, Vatican Palaces (1513–14), is just one of the many examples that show the diffusion of linear perspective in Renaissance painting. Such a mathematical construction of pictorial space is grounded on the homogeneous and continuous space of modernity. It is postulated by Alberti as a *spatium* that, Branco Mitrovic writes, 'can be depicted independently of the bodies that fill it' and whose points 'are quantifiable and geometrically definable'[17] as well as consistently representable from any point of view. At the same time, the bars of the prison window look like a demonstration of the pictorial procedures used by artists who squared their *bozzetti* to reproduce or enlarge them on panels and walls by means of scaled grids.[18] An alternative to the *spolvero* technique (pricking holes along the lines of the drawing and dusting charcoal powder through them), the *mise au carreau* also made it possible to subdivide a fresco into batches, transferring one square at a time. This operation suggests an 'assembly line'-like form of production, in which something is produced by a series of discrete but inter-related actions that can be apportioned between various operatives.[19] This also gives the grid the sense of a 'scientific' tool, still present today in archaeological or police surveys, which often use a network of wires stretched to a square wooden framework to organise the visible. Applied to a sort of 'open window', Raphael's cage also evokes instruments that were devised to visualise Alberti's veil or *reticolato* and to produce perspective views on paper that was correspondingly squared, of the sort illustrated by Albrecht Dürer in his celebrated engravings (Fig.3).[20] Added to this, the optical analogy between *perspectiva naturalis* (concerning natural vision) and *perspectiva artificialis* (concerning representations) is here emphasised. The shadows of the bars splay in relation to the light projecting from the centre of the angel's bright aura, which is both source of the light and perspectival vanishing point. At the same time, they seem to prefigure the practice of projecting grids on to walls and vaults to reproduce drawings and construct *quadratura* and anamorphic schemes upon complex surfaces in the Baroque Age.[21]

Raphael's work benefited from decades of pictorial and architectural experimentation. Perspective construction had been gradually encouraging a mathematisation and modularisation of architectural elements;[22] according to Arnaldo Bruschi, design based on architectural drawing had tended to promote a simplification of compositional rules and forms[23] – especially so in the definition of the plan. In opposition to this process, the expanding studies of antique buildings and the growing authority of ancient sources – think of Vitruvian precepts and all their interpretative problems – introduced elements of complexity[24] and favoured the persistence of proportional systems based on regulating lines and the golden ratio. An early manifestation of both these trends can be found in the work of Filippo Brunelleschi (1377–1446). He began to compose his own architecture on a square grid that allowed the positions of structures to be located on plan according to a module, but adopted Vitruvian proportions to design the elements of the architectural orders,[25] assigning to perception the task of unifying a structure already conceived in mechanical and serial terms, almost ready for the prefabrication that would arrive with the industrial age. While acknowledging that it is difficult confidently to separate interpretative inclination from historical truth – especially when dealing with a work such as Brunelleschi's innovative, modular project for the church of Santo Spirito in Florence – one might argue that the following decades of the 15th century provided a period needed to assimilate and systematise these early results and to foster squared paper as a design environment.

Fig.4 Francesco di Giorgio Martini, Plan of fortress (drawing by author, after *Codex Magliabechianus* II.I.141, f.55), with a proportional grid and the sequence of the Attackers' (A) and Defenders' (D) positions in nine stages.



The grids of Francesco di Giorgio Martini and Filarete

The transition from the abstract tool of the square grid to the material instrument of squared paper can be traced back to the treatises of Antonio Averlino, called Filarete (1400–*c*.1465), and Francesco di Giorgio Martini (1439–1501). Unlike Alberti's *De Re Aedificatoria*, which had no illustrations and trusted the reader's skill in following his elementary algorithms,[26] both Filarete and Di Giorgio interspersed the written text of their manuscripts with diagrams, schemes and drawings of architecture, machines and decorations.

Francesco di Giorgio's matrix for the design of religious buildings is generally a combination of quadrature procedures, regulating lines and square grids,[27] while many of his schemes for civic buildings are based on regular square grids. However, it is not clear how these linear diagrams – which are without structural thicknesses – would be turned into actual plans.[28] 'The proportional grid can be expressed through lines and immaterial points; but architecture is made of walls, pillars, physical elements that, as Alberti would have said, "occupy a place".'[29] Thicknesses 'not only generate inconsistencies in the relationships between solids and voids, in the structural plausibility of wall thicknesses, and in the connections between the parts, but also change the proportional relationships'.[30]

Conversely, in illustrating his military projects, Di Giorgio represented the plan of his fortresses with the actual form of the walls. The grid's use is particularly evident both in the definition of the wall thickness and in the arrangement of rooms, which turn the plan into a defensive system able to slow and disorientate the attackers (Fig.4). Although no explicit reference is given in the text, reconstructing the mazy plans of Di Giorgio's *capannato* proves that the thickness of internal walls is equivalent to one square and that of external walls is twice this – as large, indeed, as most of the internal rooms.[31] Filarete tested the use of the grid on buildings, cities and gardens.[32] He explicitly described and used square grids both as a proportional scheme and to scale a drawing, which also provided him with a simple aid in drawing lines and calculating distances. He returned several times to the use of the grid in scaling, often emphasising that, even if the 'things of the building' were designed appropriately, 'to understand' the building in the drawing requires 'thinning the intellect a little'.[33]

Filarete invoked the grid for reasons of proportion when discussing the plan of the city of Sforzinda and its churches. He used it to define the sizes of buildings in both plan and elevation, and the thicknesses of their walls. In particular, the description of the Cathedral of Sforzinda is associated with the diagram of an otherwise vacant square grid, which implicitly invites the reader to draw its plan to reproduce the project.[34] However, the reconstructions attempted over the years by John R. Spencer,[35] Liliana Grassi[36] and Jens Niebaum[37] have highlighted several inconsistencies, which can be solved only by superimposing two or three different grids.[38] This approach may be derived from Vitruvius:[39] as the Roman architect used different modules to establish proportions and dimensions of the different parts of the architectural orders,[40] so perhaps Filarete overlapped different grids to design his plans and elevations.

Drawing the urban plan of Sforzinda is even more problematic. In the text, Filarete reports the instructions for first dividing the 7,500 *braccia*-sided square (equal to 20 *stadia*) into five parts, each 1,500 *braccia* long (or four *stadia*): 'I will show you […] by drawing [*lineamento*] ['the proportional scheme and you will see it'] squared in *quadri piccoli* […] of four *stadia* for each one'.[41] But the main square is required to be divided into 375 parts to obtain the reference module, which is completely impracticable in the restricted space of the sheet of paper.[42]

Niebaum's attempts at reconstruction demonstrate that Filarete considered the square grid as a communicative, scaling, proportioning and dimensioning tool. They also reveal his difficulty in using a single grid to design complex architectures. Moreover, as the squares reduce in size in order to encompass every single element, the grid itself – while it can still work as a design tool – becomes useless for communicating (and easily reproducing) the project.

Bramante, his followers, and squared paper

Filarete's words and diagrams, known through copies of his manuscripts, are reputed to have influenced not only the reception of Vitruvius but also the *modus operandi* of Donato Bramante (1444–1514). Presumably, Bramante had become familiar with the proportioning uses of the grid in Milan around 1480, when Filarete was working at the Ospedale Maggiore, although he probably had already learned to use grids from his training as a painter and perspective designer.[43] However, only in Rome did he achieve a complete mathematisation of the plan, employ the architectural orders to mark the intersections of the grid's lines – as in the convent of S. Maria della Pace (1500–1504)[44] – and begin to use squared paper.

In 1505, while designing the Vatican Belvedere, a garden 100 x 300 metres in area that was to develop architecture on a landscape scale, Bramante was entrusted with another 'extra-large' project, the St Peter's Basilica in Rome. Some of his original plans, which are conserved at the Uffizi in Florence, show lines made with a pencil upon a squared paper.

Guido Beltramini[45] has lucidly explained Bramante's design procedure regarding one of them, the f.20A (Fig.5).[46] With pale sepia ink, Bramante traced a grid of squares, in which 60 squares ('minutes') are as long as a Roman palm (22.34cm). He then established that each of them (which has sides 0.37cm long) corresponded to a dimension of five palms (111.7cm) in the building, thus obtaining a drawing at a scale of approximately 1:300. At this point, he scaled and traced the plan of the already existing Constantinian basilica of St Peter and the choir of Nicholas V before beginning to superimpose his design. For this purpose he adopted, for the first time among architects, a sanguine pencil (red chalk) which, unlike the pen, offered lower resistance upon the paper and allowed him to draw continuously and fluently, with no interruption for tipping in the ink.[47] Beltramini has argued that precisely this technical innovation allowed Bramante to focus on the shape of the void and to make it the fluid protagonist of the project, obtaining the form of the walls as a result. From this point of view, the underlying grid – which acted as a sort of ruled design environment – therefore constituted a barely perceptible visual reference that guided Bramante's hand and channelled its trajectories.

A square grid is present in other variants of the design for St Peter's at the Uffizi, such as f.7945A (where the grid is subdivided according to increments of 48 rather than 60),[48] while it is missing in the famous Parchment Plan,[49] the presentation half-plan that has given birth to many interpretations.[50] Although Hubert[51] overlaid a grid based on subdivisions of 60 to demonstrate that even the Parchment Plan largely fits it, it occasionally differs and gives rise to alternative proportional hypotheses. Tim Benton, for example, has identified a spatial and modular system composed of the large Greek cross of the central naves (120 palms wide) and four minor Greek crosses in the corners (60 palms wide), which largely complies with the grid, although it is contradicted by the central pillars and the detailed forms of the many niches.[52] This, like other interpretative attempts, seems to show that Bramante's project is based on complex geometric operations, the outcomes of which could be comprehended only by a very dense grid. At the same time, the grid allowed Bramante to pursue what Trachtenberg defined as 'seemingly innumerable multicellular, multilayered, and multileveled spaces around the central domed void' according to an 'unstable scheme [that] had the intrinsic tendency to fragment and multiply in fractal-like levels of complexity.'[53] In this sense, the grid could be used to relate explorative sketches of singular elements of the project, ranging in scale from the polygonal pillars to the general plan.

While Bramante's squared paper always represents the unit of measurement chosen and manifests the scale of reduction, it also provides an abstract visual reference that symbolically demonstrates that the curvilinear plan of St Peter's, which could be mistaken for a decorative pattern or a parterre in a garden, was anchored in a rational process.

Bramante taught the use of the grid to his assistants: first Antonio da Sangallo the Younger (1484–1546), who helped him on the designs for St Peter's;[54] then Baldassare Peruzzi (1481–1536), formerly a painter and *prospettico* ('perspectivist'). Together with Giovanni Battista da Sangallo, called il Gobbo (1496–1548),[55] they produced a group of drawings on squared paper, now in the Uffizi. All of them are plans and, in some cases, present surveys of ancient buildings, mainly drawn by Antonio[56] and by Baldassare.[57] Some of these drawings might have been made in a limited time frame, given the similarity and the closeness of the sites. The lack of measurements suggests that they resulted from a redrawing procedure, almost a restitution *in pulito* from other sheets of notes and measurements. Alternatively, they could result from a sort of experimental rapid surveying of antiquities, in which the squares allow the restitution of the measured elements directly to scale and automatically rectify any irregularities. The latter hypothesis seems closer to the vast cultural project of Raffaello Sanzio (1483–1520). As known, after Bramante's death in 1514, Raphael intended to systematically survey and represent Roman antiquities in orthogonal projections.

Raphael also shared Bramante's method of design and his way of painting the sectioned part of the plan. Two plans of the Chigi Chapel drawn by Raphael and Antonio on a squared sheet are dated to 1513. In this case, the back wall's length of 15 2/3 palms almost corresponds to 16 squares in f.165A and 32 squares in f.169A. In the former sheet, a small square with sides 0.7cm is equal to a palm (22.34cm), according to a scale of 1:32, while in the latter, the module is half-palm.

Most of the design drawings of this group show important pre-existing elements, too. For example, the project for the church of S. Francesco a Ripa[58] derives from the survey of the existing church, with half the plan showing the building *ante-operam* and the other half *post-operam*.

The project for the Palazzo Orsini[59] was designed by Baldassare in collaboration with Antonio for the sons of the Count of Pitigliano around 1518–19.[60] Made of four squared sheets (each of the small squares represents two palms) fixed together, it shows an accurate plan of a large palace around the circular remains of the *calidarium* of the ancient Baths of Agrippa by the Pantheon, with a rectangular courtyard and a number of secondary ones (Fig.6). All the rooms are described with their function, size, doors and windows, and some are equipped with baths and fireplaces. Most of the plan is delineated in ink but some constructions and alternative solutions are in lapis and red chalk. While existing structures are registered by pink ink, the new ones are filled with sepia and aligned with the lines of the grid, with a few apparent exceptions (Fig.7).

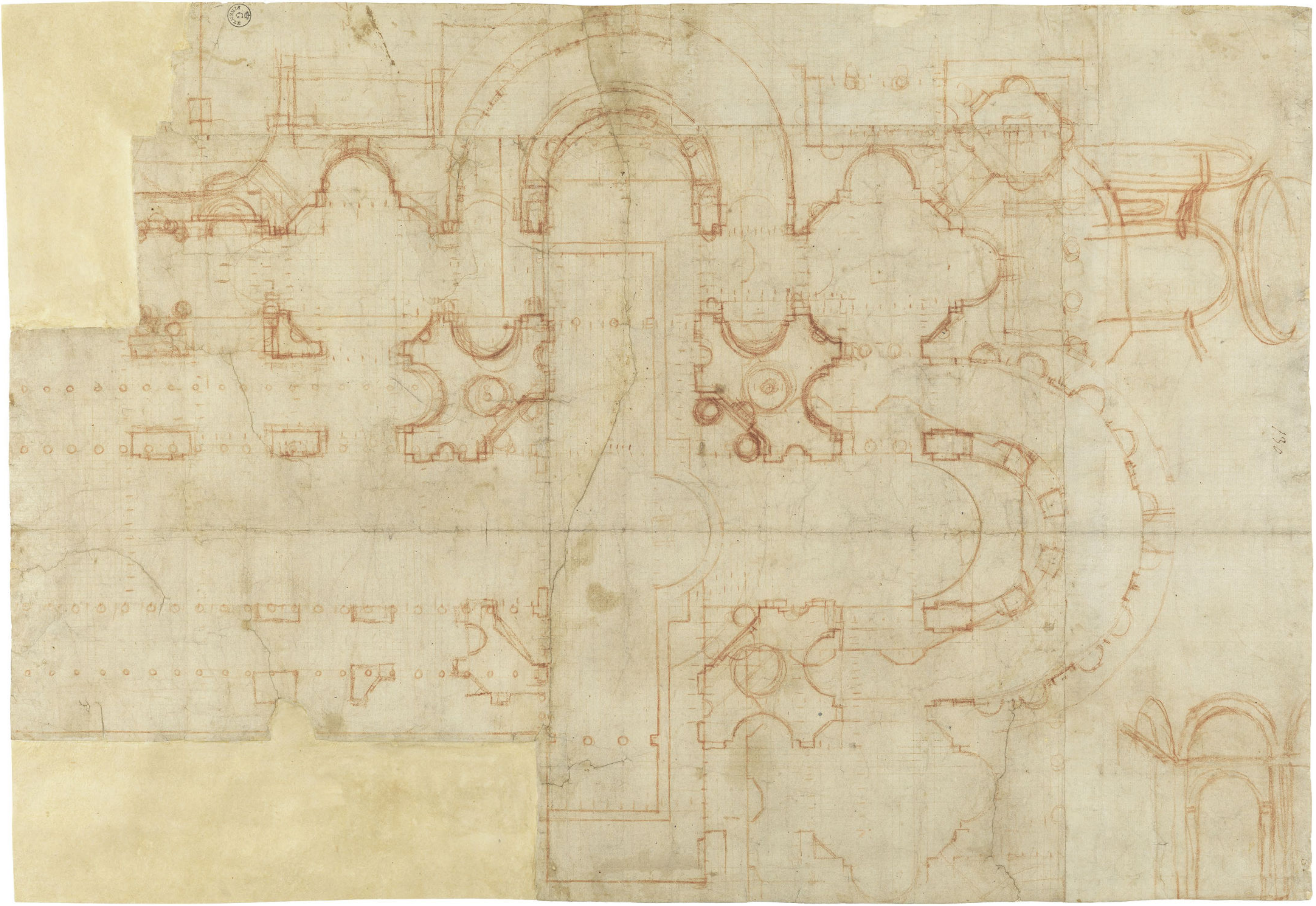
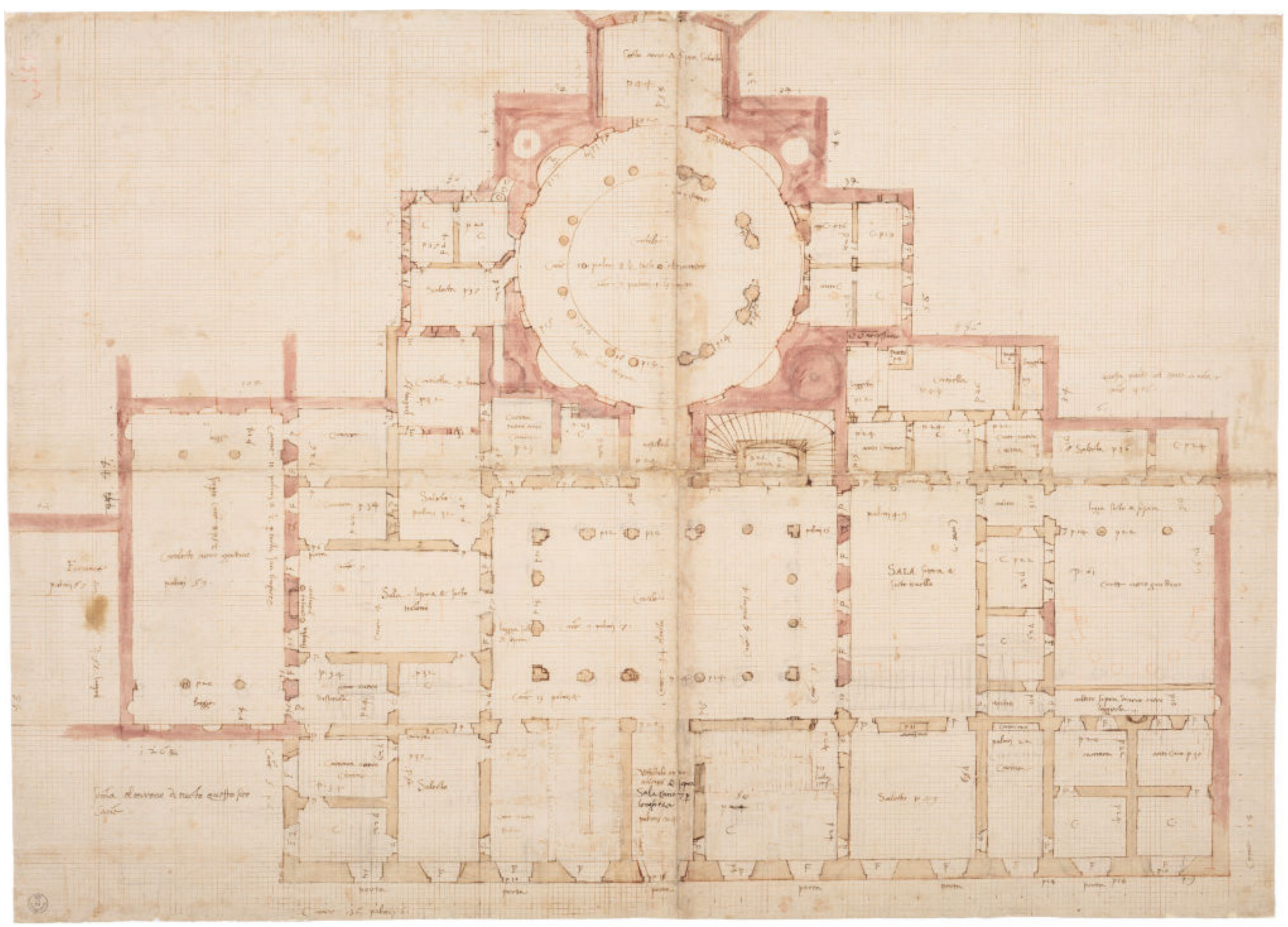
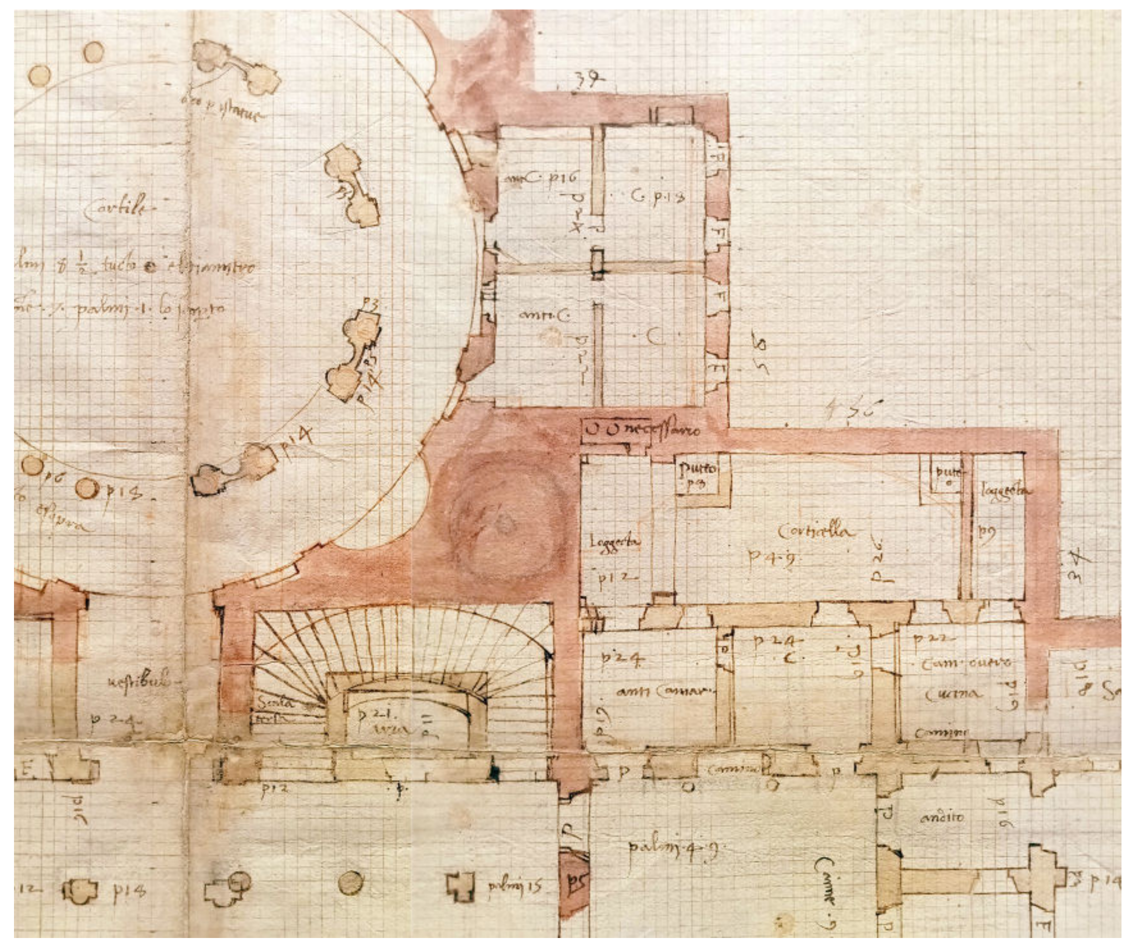


Fig.5 (Previous) Donato Bramante, Plan study for the Basilica di San Pietro, Rome, 1506. Red chalk on squared paper, 47.2 x 69cm. Gabinetto Disegni e Stampe degli Uffizi, Florence, f.20A. Reproduced with the permission of the Ministry of Culture.

Fig.6 Baldassare Peruzzi, Plan for Palazzo Orsini, *c*.1519. Gabinetto Disegni e Stampe degli Uffizi, Florence, f.456A. Ink, red chalk and watercolour on squared paper, 57.8 x 80.7cm. Reproduced with the permission of the Ministry of Culture.

Fig.7 Baldassare Peruzzi, Plan for Palazzo Orsini, *c*.1519. Gabinetto Disegni e Stampe degli Uffizi, Florence., f.456A (detail). Ink, red chalk and watercolour on squared paper, 57.8 x 80.7cm. Reproduced with the permission of the Ministry of Culture.





Toward a grid-based design system

After Sforzinda, the Sienese Pietro Cataneo (1510–1569/73) – a pupil of Baldassare Peruzzi, like Serlio himself – was the first to publish designs for fortified cities on grid patterns.[61] Parallel with this, a vertical use of the grid as a system to establish sectional proportions emerged in the *Premier tome* of the French architect Philibert de l'Orme (1514–1570).[62] In particular, the section of a three-nave church[63] demonstrates the transition from a scheme of regulating lines (derived from Di Giorgio, or perhaps an earlier source[64]) into a seven-square diagram that is 'modulated into a grid on the intersections of which all the main measures are determined'.[65] Vertical grids had already appeared on the pages of the commentary on Vitruvius published in 1521 by Cesare Cesariano, a follower of Bramante, where they were used to demonstrate the proportions of the human body. His Vitruvian-derived *Homo Corporis Mensura* is drawn on a grid of 30 x 30 squares, half of Bramante's 60 x 60 grid, even if some elements are elusive and require additional graphic devices and units of measurement. His *Homo Ad Circulum et Ad Quadratum* (Fig.8) shows, on the other hand, a use of the square grid that is principally symbolic. As Veronica Riavis has noted, 'the grid has more representative utility than metric, as it is independent from the graphic scale reported by the author'.[66]

Added to this, in Cesariano's *Cavaedii Tuscanici Figura*[67] the canonical chequered floor of perspective views is extended to the walls (Fig.9). Although only partially serving to size and place doors and windows on the façades, the grid's lines highlight the relationship between the horizontal and vertical planes and consequently imply a mathematisation of architecture. While Sebastiano Serlio used the grid in his *Second Book* to demonstrate perspective compression and acceleration on the slopes of the theatrical scenes,[68] the Florentine engineer Bonaiuto Lorini (1540–1611) endorsed his *profilo graticolato* (grilled profile), with a square corresponding to one step, as a procedure to quickly measure and restore the shape of the fortifications as well as to describe the inclinations of walls (Fig.10).[69]

In this proliferation of applications, grid-based systems also affected garden design, a new field of architectural experimentation. As the paper drawing surface promoted the meeting and cross-pollination of different visual arts, scales and techniques, so the garden became the place where territorial and urban planning met with hydraulics, botany, architecture, antiquity, decoration, and many more concerns. An overarching design system was required to orchestrate functions and perspectives and to organise the work of different operators, and the square grid was seemingly able to provide this.

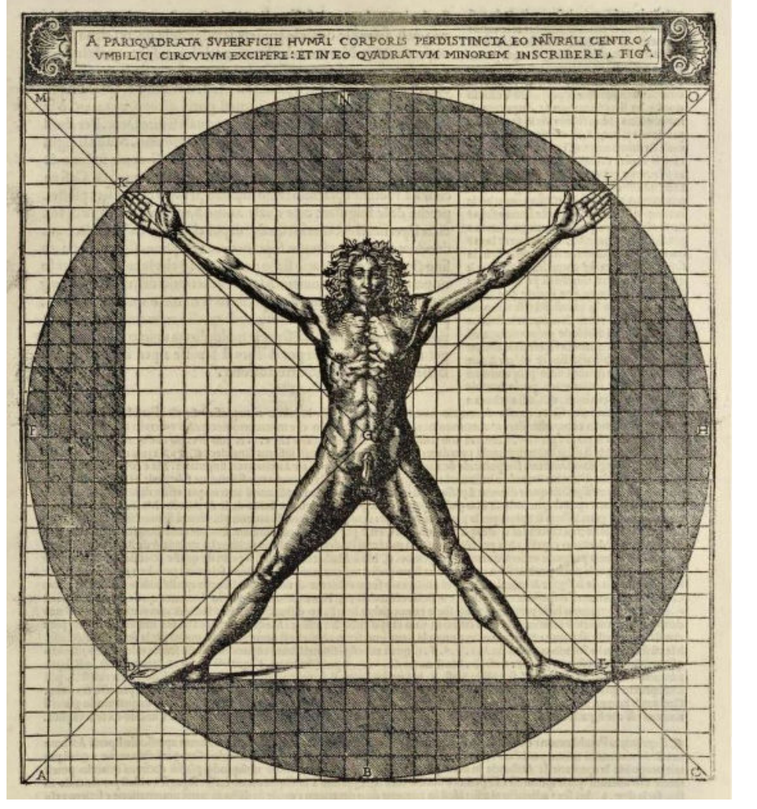
In this sense, garden labyrinths can be viewed as an early manifestation of grid-based design. Already widespread in Roman floor mosaics and medieval cathedrals, labyrinths were utilised during the Renaissance to express the idea of protection and were consequently associated with weapons and fortresses, as testified by the literary works of Giovanni Fontana,[70] Di Giorgio or Francesco de' Marchi.[71] At the same time, they became a recurring element in Italian gardens, both as the emblem of Daedalus' skills and as a figure of mediation with myth and antiquity. The early labyrinths designed by Italian architects – Di Giorgio drew two, Filarete five, Serlio two, Giulio Romano some for the court of Mantua – are irregular structures, most of which were presumably to be built of wooden planks covered with climbing plants, as documented in Rome and Tivoli. Their walls are generally thinner than the corridors, which may vary in width.[72]

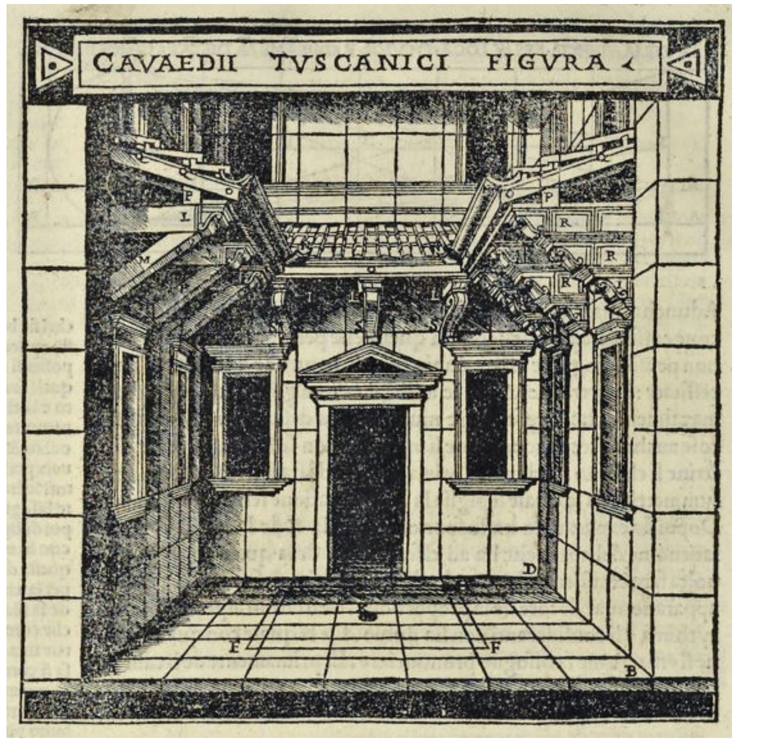
Conversely, the vegetal labyrinths designed by the Flemish architect Hans Vredeman de Vries (1556–1604), who derived much of his architectural knowledge from Serlio's books, were designed on a regular square grid, with corridors as wide as the walls. One of the

Fig.8 Cesare Cesariano, *Homo Ad Circulum et Ad Quadratum*, from *Di Lucio Vitruvio Pollione de Architectura Libri Dece* [...] (Como: Gottardo de Ponte, 1521), f.50r.

Fig.9 Cesare Cesariano, *Cavaedii Tuscanici Figura*, from *Di Lucio Vitruvio Pollione de Architectura Libri Dece* [...] (Como: Gottardo de Ponte, 1521), f.96v.

Fig.10 (Overleaf) Bonaiuto Lorini, Section according to a *profilo graticolato*, from *Delle Fortificationi di Buonaiuto Lorini Libri Cinque*, I (Venice, 1597), 30–31.





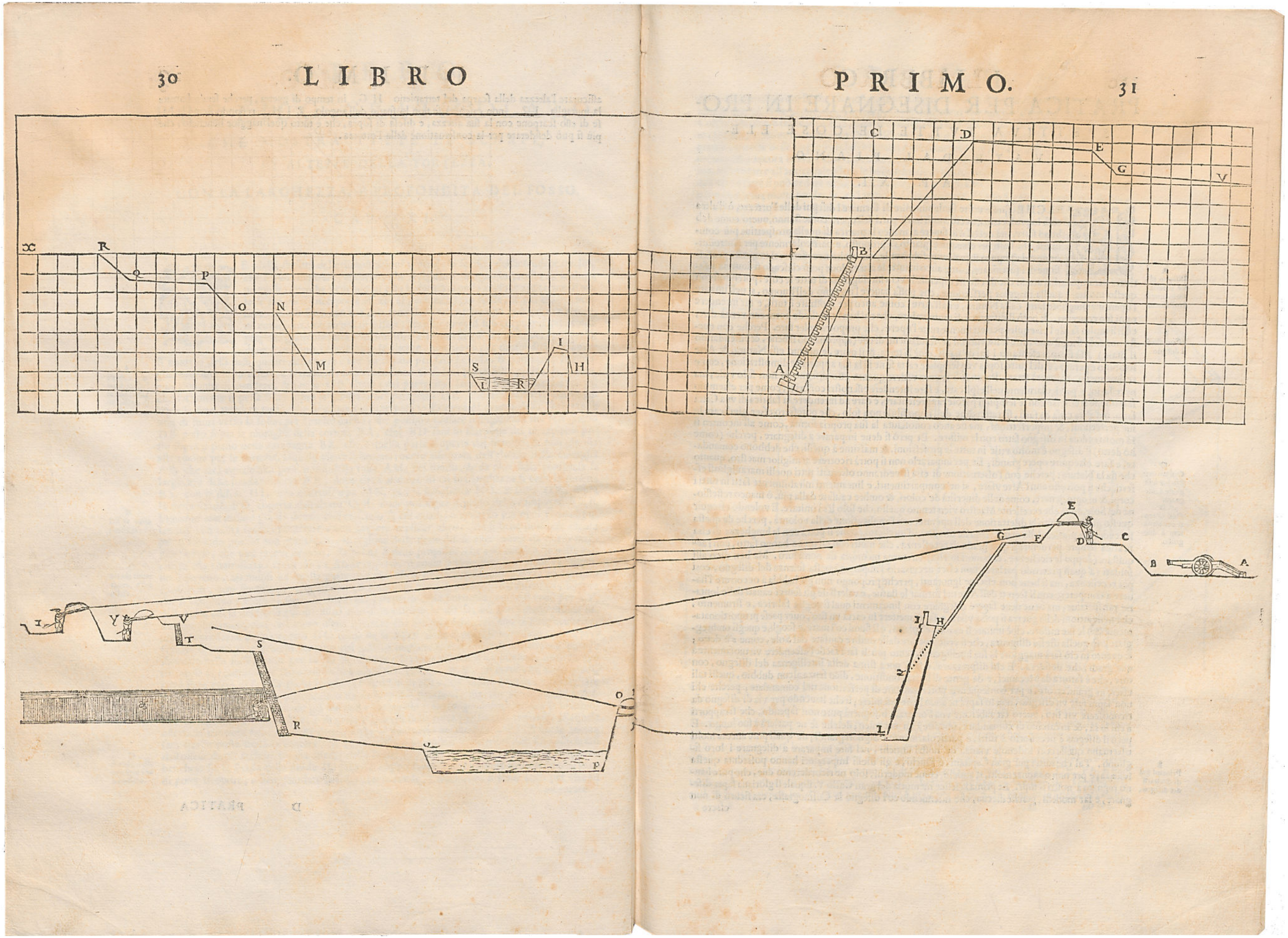

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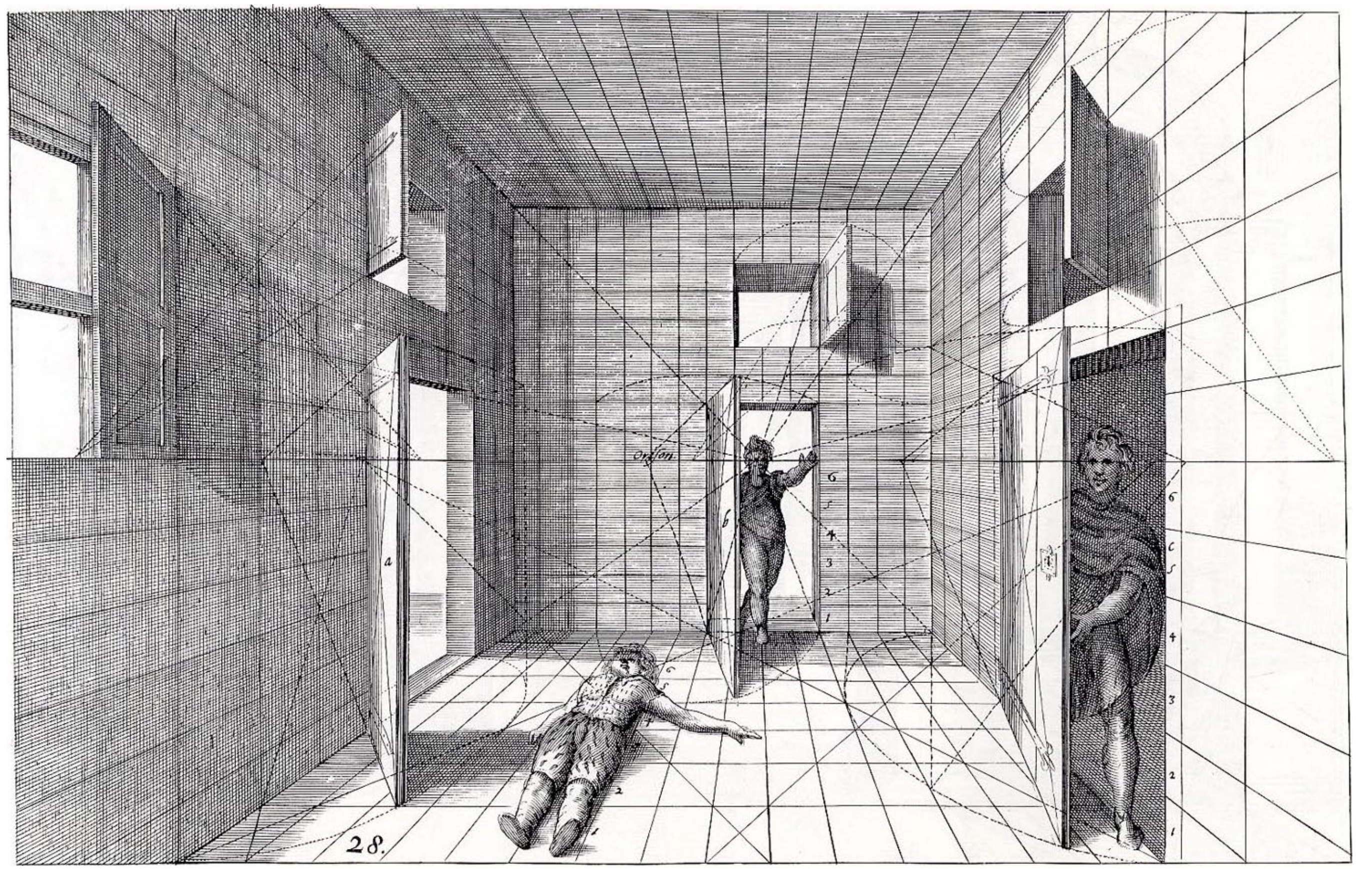
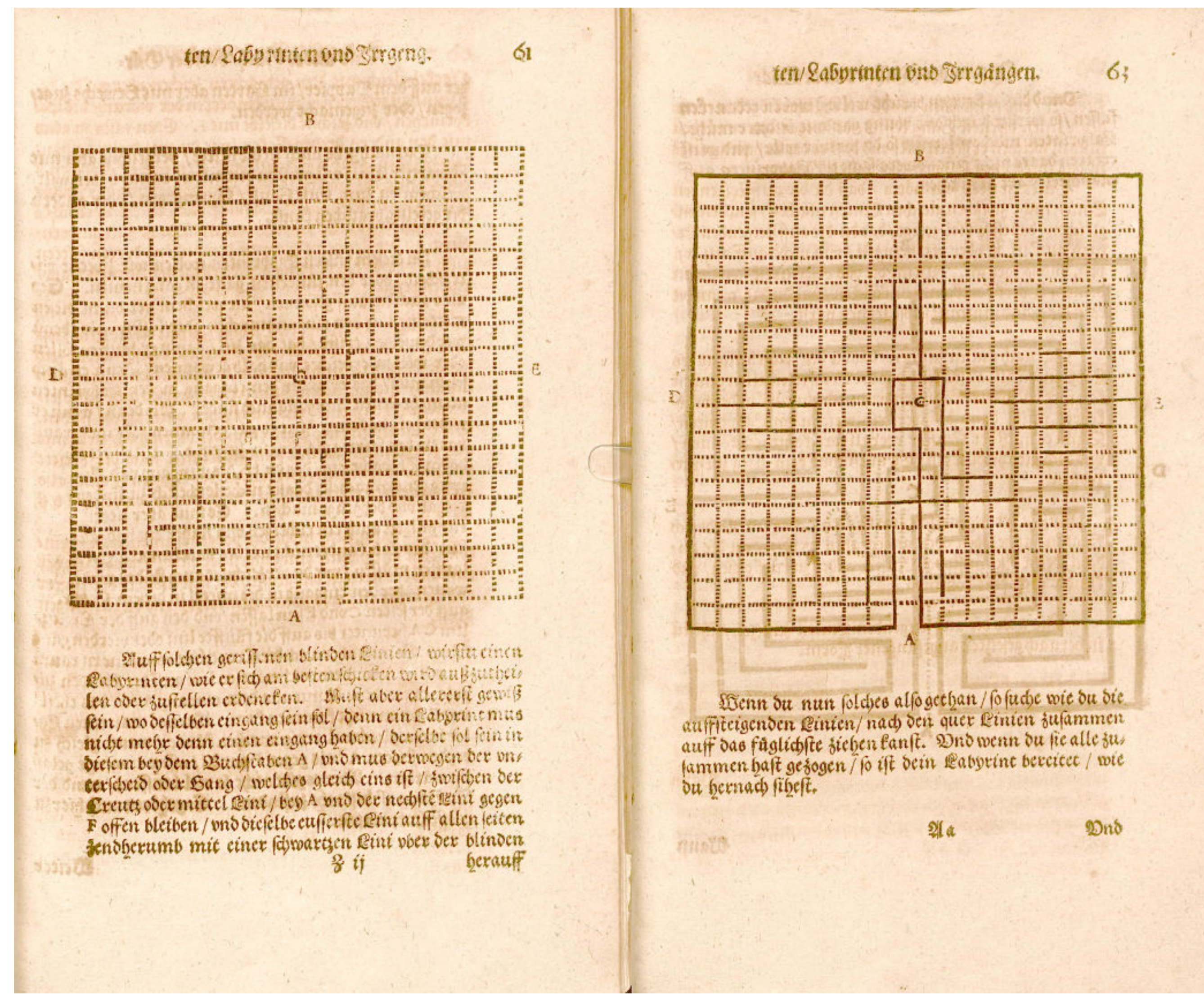
Fig.11 Hans Vredeman de Vries, Cubic room in perspective, from *Perspective c'est à dire, le très renommé art du poinct oculaire* (Lugduni Batavorum, 1604–1605), pl.28.

Fig.12 Daniel Loris, How to draw a labyrinth on a square grid, from *Le Thrésor des parterres de l'univers: contenant les figures et pourtraits des plus beaux compartimens, cabanes, et labyrinthes des jardinages* (Geneva: Estienne Gamonet, 1629), 61, 63.





main exporters of Italian architecture to Northern Europe, Vredeman de Vries was a polymath, producing books on orders, ornaments, antiquities, architecture, urban views, perspective, and – of course – gardens.[73] The cubic room illustrated in his perspective treatise looks like a manifesto proclaiming the homogeneity of space, in which the invisible grids that cover the floor, ceiling and walls, regulating the form and position of the openings, are revealed (Fig.11).[74]

Not only did church-floor labyrinths in the Netherlands have corridors and walls made with square tiles of the same size but different colours,[75] but the garden labyrinths designed by Vredeman de Vries and his followers are rigidly based on regular square grids. In the 1597 book *Gartem Ordnung*, the German Lutheran pastor Johann Peschel proposed a grid of 18 × 18 squares and dedicated several pages to describe carefully how to make a labyrinth out of it, according to one of Serlio's designs.[76] In his *Le Thrésor des parterres de l'univers* (1629), Daniel Loris proposed a grid to realise a labyrinth with the walls twice as thick as the corridors (Fig.12).[77] Garden designers such as Daniel Marot and Johann Herman Knoop were first referred to as 'mathematicians'.[78] Shared by major architects, their science, which was 'a combination of survey, horticulture, and mathematics',[79] contributed to the definition of a multi-scale, grid-based design system that intrinsically presented an alternative to Vitruvian dispositions.

The early architectural consequences of these applications can be found in the works of a small group of Dutch architects, active in the mid-17th century, whose work has been scrutinised by Konrad Ottenheym. While some of them, like Jacob van Campen or Pieter Post, demonstrated a generic interest in Italian Renaissance architecture, others pursued geometric methodologies to simplify the complex architectural formulae presented in Italian treatises into mathematical applications of the grid that could still carry ideas of universal harmony. This can be seen in Nicolaus Goldmann's cubic villa, in Philips Vingboons' ideal villa, and in Adriaan Dortsman's designs for the Town Hall of Maastricht or Finspång Castle. These are all projects whose drawings – as Ottenheym puts it – display the use of 'arithmetical grid systems as well as geometrical constructions to achieve unity and coherence within their designs'.[80]

Considerations

During the Renaissance, grids primarily found their early applications as proportional systems in specific fields, such as military architecture and garden design, which could not be addressed by antique models and required an interdisciplinary, pragmatic approach. Leaving regulating lines aside, Di Giorgio adopted the grid to design his labyrinthine fortified buildings and to dimension both voids and wall thickness; Cataneo deviated from the tendency to identify Vitruvius' urban descriptions according to radial (and cosmological) schemes and extended the grid to the whole fortified city; Lorini rotated the grid on the vertical to manage the profile of bastions and the castle plans at different levels. Through these steps, too, military architects were to depart from the sphere of Vitruvian influence,[81] to acquire new competences, to surround themselves with mathematicians, politicians, medics, adventurers, and topographic draughtsmen working as spy-painters,[82] and to develop innovative representations, such as the *prospettiva soldatesca* (axonometric view) or other *figure di dimostrazione*.[83]

Although the ancient sources and models can be mainly considered as an obstacle to this process of mathematisation, Hans Hubert has conjectured that Filarete and Bramante derived the idea of using a square grid as a proportioning system from original Latin sources such as the notes of the Roman soldier and civil engineer Sextus Iulius Frontinus and the land surveyor Siculus Flaccus about *centuriatio*.[84]

The 6th-century manuscript of the *Corpus Agrimensorum Romanorum*[85] in Wolfenbüttel presents many diagrams with square grids applied to land and city planning (Fig.13) and some of them show affinities with Filarete's figures. In this regard, the many labyrinths Filarete designed in his treatise – five, as previously mentioned, and all square – could be interpreted as evidence of this legacy. Whilst a combination of grids was needed to proportion the parts in plan and elevation, a square grid helped express the scale of reduction and worked as a pedagogical, communicative tool that allowed the reader to sketch the project and to establish a rational line of communication between the architect and the kind of enlightened client Filarete expected to encourage.

Presumably inspired by ancient practices of land division, Filarete also adopted grids as a flexible system to design large buildings such as the Milan hospital, or the whole city of Sforzinda. Similarly, the pioneering design experience with the extra-large size of the Vatican Belvedere and St Peter's Basilica might have convinced Bramante that he should test squared paper in surveying and designing architecture. Though it was probably adopted as a modulated background onto which existing structures could be traced and the plan of the garden or the church drawn freehand, squared paper was revealed as a powerful, universal tool to facilitate exchanges with his close collaborators, and to approve and manage their contributions.

While Cesariano oscillated between a metric and a symbolic use of the grid, expressing both the measure and the invisible relationships that bind the parts, Antonio da Sangallo, Giovanni Battista da Sangallo and Baldassare Peruzzi used squared paper to survey monuments and design plans that match the pre-existing elements and the new constructions according to a module, as can also be seen in the designs of Raphael himself. In particular, Peruzzi's plan for the Palazzo Orsini shows a mature and aware use of squared paper as a tool for calculating the proportions and dimensions of architecture. Conjecturally, such a squared plan can also refer to his practice in perspective drawings, providing a geometrical support to create, for example, a bird's-eye view of the palace.

Hoever, it is difficult to write anything definitive about the group of drawings at the Uffizi. Hubert stressed that the phenomenon of squared paper is limited to about 30 years (1506–1536) and to the area of Rome.[86] Wolfgang Lotz has argued that 'squaring the sheets was … an annoying and tiring procedure', and it is therefore 'no coincidence that the group of these drawings is small in number. The squared paper experiment had a short duration and was soon abandoned',[87] presumably in favour of the graphic scale and the practice of tracing fundamental lines with a metal point to guide the actual drawing.

The diffusion of gardens by palaces and villas in the 16th century gradually promoted the recovery of the *Agrimensorum*'s approach and its harmonization with architectural design procedures. In the late 16th century, Vredeman de Vries linked the design method of land planners and garden designers with the practice of architects, indirectly also promoting the use of the grid to compose buildings. According to the geometric suggestions provided by labyrinths, the grid proved to be a rational, scientific and generally applicable instrument of design. However, its ability to replace hybrid systems derived from the interpretation of Vitruvius and ancient buildings was recognised slowly in the 17th century (and at first outside Italy), when philosophers, scientists and architects began to distance themselves from the pre-established authority of ancient writers. Copernicus and Galileo, for example, boosted the process of 'geometricisation' of lived space, supplanting visible reality with a world of abstractions, relations and equations whose language is mathematics,[88] contributing to turning perspective from a symbolic figuration into a tool for visualisation.

Sometimes, this distance emerged properly in the new editions of Vitruvius.[89] For example, in his edition of *De Architectura*,[90] Claude Perrault denied the *a priori* status of given harmonic relationships and the need to derive proportions from the human body, questioning the authority of Vitruvius and legitimising research and experimentation into new criteria of proportion. But it was only in the 18th century, after the definitive formulation of 'descriptive geometry', that drawing lost its symbolic residue and the grid turned into a fully abstract representation of inhabitable space – a 'practical device for providing simple rules for determining the proportions and locations of rooms, doors, and windows. No longer a network of invisible lines to elucidate architectural meaning, the grid became a mere instrument for simplifying the design process.'[91]

Conclusions

As children, we use squared paper for arithmetic, because squares help to isolate the individual digits and put them in rows and columns. We use it to draw geometric flowers or butterflies by tracing lines along the edges of the squares and filling them in with colours, like pixels on the screen. And we also use it to play games like Battleships, in which the Cartesian grid is deployed to establish the position of the vessels through two co-ordinates shared by distant sheets. These functions – to isolate or to differentiate, to modulate, to size, to place, to share and transfer – were also known to Renaissance architects. Yet, the architectural drawings that show a grid pattern in the 15th and 16th centuries are very few. The main reason seems to lie, rather than in the effort of preparing the squared paper, in the implicit resistance offered by other design systems which, although more complex, carried greater authority, either because they referred to exemplary buildings or were supported by ancient literature.

In parallel to civil and sacred buildings, architects used square grids to simplify and mathematise compositional procedures in military architecture and gardens. Alternative or complementary to Vitruvius, they were required to orchestrate large, multi-disciplinary projects in direct relationship with the natural territory. The diffusion of square labyrinths and their direct connection with antiquity and architecture offers a solid clue here.

While the grid constituted an instrument shared by artists engaged in different fields and disciplines, from forms of perspectival representation to the design of decorative patterns, it is likely that it was the spread of larger and larger gardens which most significantly contributed to its further development and diffusion in architecture as a generalisable approach to design – a tendency that would also in due course entail the extension of the grid to vertical surfaces. And notably this happened first in Northern Europe, where Vitruvius' voice was softer.

1 Bernardo Vittone, *Istruzioni Elementari per Indirizzo de' Giovani allo Studio dell'Architettura Civile: Divise in Libri Tre* (Lugano: presso gli Agnelli, 1760).
2 Alberto Pérez-Gómez, *Architecture and the Crisis of Modern Science* (Cambridge, MA: MIT Press, 1983), 109.
3 Nicola Carletti, *Istituzioni d'Architettura Civile* (Naples: Stamperia Raimondiana, 1772).
4 Peter Collins, 'The origins of graph paper as an influence on architectural design', *Journal of the Society of Architectural Historians*, vol.21, no.4 (1962), 159–62.
5 Jacques Guillerme, *La Figurazione in Architettura* (Milan: Franco Angeli, 1982), 107.
6 Luke Howard, 'On a periodical variation of the barometer, apparently due to the influence of the sun and moon on the atmosphere', *Philosophical Magazine*, 7 (1800), 355–63 (357).
7 Frederick William Herschel, 'On the investigation of the orbits of revolving double stars', *Memoirs of the Royal Astronomical Society*, 5 (1832), 171–222 (171–72).
8 See Fabio Colonnese, 'Transparent paper as a modernist architectural design environment', in *Digital Modernism Heritage Lexicon*, eds Cristiana Bartolomei, Alfonso Ippolito and Simone Tanoue Vizioli (Cham: Springer, 2020), 57–79.
9 John Perry, *England's Neglect of Science* (London: Unwin, 1900), 30. See also 50–56.
10 See Lev Manovich, *Software Takes Command* (New York: Bloomsbury Academic, 2013).
11 I thank Mark Dorrian for this suggestion.
12 Friedrich Nietzsche, *Epistolario (IV)*, trans. M.L. Pampaloni Fama and M. Carpitella (Milan: Adelphi, 2004), 163.
13 According to Panofsky 'the Egyptian network does not have a transferential significance, but a constructional one, and its usefulness extended from the establishment of dimensions to the definition of movement'. Erwin Panofsky, *Meaning in the Visual Arts* (Garden City, N.Y.: Doubleday Anchor Books, 1955), 60–1. See also: Yoshifumi Yasuoka, 'A new interpretation of the grid system reform in the Late Period', *The Journal of Egyptian Archaeology*, vol.107, nos 1–2 (2021), 265–80; Whitney Davis, 'Scale and pictoriality in ancient Egyptian painting and sculpture', in *To Scale*, eds Joan Kee and E. Lugli (West Sussex: John Wiley & Sons, 2016), 26–43.
14 Marco Frascari, 'A reflection on paper and its virtues within the material and invisible factures of architecture', in *From Models to Drawings: Imagination and Representation in Architecture*, eds Marco Frascari, J. Hale and B. Starkey (London: Routledge, 2007), 23–33.
15 Johann Schönsperger the Younger, *Ein New Getruckt Model Büchli* (Augsburg, 1529). Metropolitan Museum of Art, New York: 18.66.2.
16 Riccardo Migliari and Marta Salvatore, 'Il "teorema fondamentale" del "De Prospectiva Pingendi"', in *Le Teorie, le Tecniche, i Repertori Figurativi nella Prospettiva d'Architettura tra il '400 e il '700*, eds Maria Teresa Bartoli and Monica Lusoli (Florence: Florence University Press, 2015), 3–23. Over the years, artists learnt how to measure, represent, and report images on every kind of surface.
17 Branco Mitrovic, 'Leon Battista Alberti and the homogeneity of space', *Journal of the Society of Architectural Historians*, vol.63, no.4 (2004), 424–39 (435).
18 Architects also used this technique. For example, Giovanni Antonio Dosio squared some of his sketches of monuments of Rome to transfer them into final drawings. See Franco Borsi, Cristina Acidini, Fiammetta Mannu Pisani and Gabriele Morolli, *Giovanni Antonio Dosio. Roma Antica e i Disegni di Architettura agli Uffizi* (Rome: Officina, 1976), 50, 55, 69.
19 Still in the 18th century, the Dutch painter Gaspard van Wittel used the grid to enlarge and transfer the palimpsest of sketches traced in the *camera obscura* on to the canvases, prefiguring a proto-industrial process of production able to create dozens of versions from a single *disegno preparatorio*. See Marco Carpiceci and Fabio Colonnese, 'Rome from behind. Notes on Gaspar van Wittel's *vedute* of the Tiber for *camera optica*', in *Disegno e Città. Cultura Scienza Arte Informazione*, eds Anna Marotta and G. Novello (Rome: Gangemi, 2015), 477–82.
20 Albrecht Dürer, *Underweysung der Messung, mit dem Zirckel und Richtscheyt, in Linien, Ebenen und Gantzen Corporen* (Nuremberg: Hieronymus Andreae, 1525), 2v-3r.
21 Marta Salvatore, 'Perspective ingenuity: methods and tools for the construction of applied perspective', *Disègno*, 6 (2020), 95–108.
22 Wolfang Jung and Paola M. Poggi, 'La prospettiva in pittura, architettura e scenografia, 1500–50', in *Storia dell'Architettura Italiana. Il Primo Cinquecento*, ed. Arnaldo Bruschi (Milan: Electa, 2002), 562–75.
23 Arnaldo Bruschi, 'L'architettura a Roma negli ultimi anni del pontificato di Alessandro VI Borgia e l'edilizia del primo Cinquecento', in Bruschi, *op. cit.*, 15.
24 An example is the contradictory reception of some apparent irregularities of the Pantheon. In their drawings, Francesco di Giorgio Martini, Antonio da Sangallo the Younger and Sebastiano Serlio corrected the vertical misalignment between order and pillars of the drum, which in their eyes had to appear to be a violation of the ancient code. See Tod A. Marder, 'The Pantheon in the seventeenth century', in *The Pantheon from Antiquity to Present*, eds Tod A. Marder and Mark Wilson Jones (Cambridge, MA: MIT Press, 2015), 304–7.
25 Gabriele Aroni, 'Vitruvian proportions in the design of the architectural orders of the Basilica of San Lorenzo', *Annali di Architettura*, 31 (2020), 7–21. Aroni's analysis reports the existence of a general system based on a square and root-two rectangle and a secondary system based on the column module and Vitruvian proportions.
26 See Mario Carpo, 'Drawing with numbers: geometry and numeracy in early modern architectural design', *Journal of the Society of Architectural Historians*, vol.62, no.4 (2003), 448–69.
27 Florence, Biblioteca Nazionale, *Codex Magliabechianus* II.I.141, fol.42. See Francesco di Giorgio Martini, *Trattati di Architettura Ingegneria e Arte Militare*, ed. Corrado Maltese (Milan: Il Polifilo, 1967), II, pl.235.
28 Marco Carpiceci and Fabio Colonnese, 'The quincunx as architectural structure. Geometry and digital reconstructions after Leonardo da Vinci's centralized plan temples', in *ICGG 2018: Proceedings of the 18th International Conference on Geometry and Graphics*, ed. Luigi Cocchiarella (Cham: Springer, 2018), 1907–18.
29 Arnaldo Bruschi, *Bramante* (Rome/Bari: Laterza, 1985), 119.
30 Francesco Paolo di Teodoro, 'Leonardo da Vinci: the proportions of the drawings of sacred buildings in Ms. B, Institut de France', *Architectural Histories*, vol.3, no.1 (2015), 1–10 (2).
31 See Marco Carpiceci and Fabio Colonnese, 'Labyrinth as passive defense system: an analysis of Renaissance treatise of Francesco di Giorgio Martini', in *FORTMED 2018. Defensive Architecture of the Mediterranean, Vol. VII*, eds Anna Marotta and R. Spallone (Turin: Politecnico di Torino, 2018), 303–10.
32 See Hans Hubert, 'In der Werkstatt Filaretes: Bemerkungen zur Praxis des Architekturzeichens in der Renaissance', *Mitteilungen des Kunsthistorischen Institutes in Florenz*, 47, 2–3 (2003), 311–344.
33 'Assottigliando un poco l'intelletto.' Antonio Averlino detto Filarete, *Trattato d'Architettura*, eds Anna Maria Finoli and L. Grassi (Milan: il Polifilo, 1972), 536. On this topic, see also 165, 180, 192, 255, 272, 292, 314–15, 333, 390, 408, 502, 652.
34 *Codex Magliabechianus*, Florence, Biblioteca Nazionale Centrale, Ms. II.I. 140, f.47r.
35 John R. Spencer, 'Filarete and central-plan architecture', *Journal of the Society of Architectural Historians*, vol.17, no.3 (1958), 10–18 (11); and 'The dome of Sforzinda cathedral', *Art Bulletin*, 41 (1959), 328–30 (329).
36 Filarete, *op. cit.,* I, Figs 9, 17, 18, 19–26.
37 Jens Niebaum, 'Filarete's designs for centrally planned churches in Milan and Sforzinda', *Arte Lombarda*, vol.155, no.1 (2009), 121–38.
38 Trying to reconstruct the hospital church in plan and elevation (on a sheet of squared paper, of course), Niebaum highlighted the existence of two different grids, one based on modules of 6 br. and the other, used mainly for the heights, on modules of 10 br. *Ibid.*, 124.
39 In the Cathedral plan, Filarete starts from a square whose side is 150 *braccia* (br) long. This is divided into 15 *quadri*, *quadretti* or *parelli* (a term derived from Alberti) of 10 br and each of them is then divided into 10 parts. Later, the main square is instead divided into nine 50 br-large squares and then into 36 smaller squares, each 25 br-sided. Although Niebaum highlighted the existence of a second grid with a module of 22 br to include the thickness of the walls, inconsistencies remain, both in the relationship between description and illustration and in some measures, which seem to contemplate the existence of a third grid. Niebaum, *op. cit.,* 128–29.
40 Mario Carpo, *L'Architettura dell'Età della Stampa* (Milan: Jaca Books, 1998).
41 Filarete, *op. cit.*, I, 62.
42 Danilo Samsa, '"Verba" e "picturae" di "lineamenta" e di "res gestae" nel Filarete', *Humanistica*, 7 (2013), 159–207 (173, n.34).

43 Filippo Camerota, 'Bramante "prospettivo"', in *Donato Bramante (1444–1514). Ricerche, Proposte, Riletture*, ed. Francesco Di Teodoro (Urbino: Accademia Raffaello, 2001), 19–46.
44 Bruschi, *Bramante*, *op. cit.*, 119–20.
45 Guido Beltramini, 'Bramante: Five Dots', Drawing Matter: https://drawingmatter.org/bramante-five-dots/ [accessed 17.10.23].
46 Gabinetto Disegni e Stampe degli Uffizi (GDSU), Florence, f.20A.
47 A century later, Carlo Maderno and Francesco Borromini will introduce graphite in architectural drawing with the effect of making tracing even more fluid.
48 See Hans Hubert, 'Ipotesi sul metodo progettuale di Bramante', in *I Disegni d'Archivio negli Studi di Storia dell'Architettura. Atti del Convegno* (Naples, 12–14 June 1991), eds Giancarlo Alisio *et al*. (Salerno: Mondadori Electa, 1994), 14–19.
49 GDSU, f.1A.
50 For example, see Michael Young, 'Paradigms in the pochè', in *Black Box, 107th ACSA Annual Meeting Proceedings,* eds Amy Kulper, G. La and J. Ficca (USA & Canada: ACSA, 2019), 190–95.
51 Hubert, 'Ipotesi', *op. cit.*, 17–18.
52 See Tim Benton, 'Bramante and the sources of the Roman High Renaissance', in *Locating Renaissance Art*, ed. Carol Richardson (New Haven: Yale University Press, 2007), 251–90.
53 Marvin Trachtenberg, *Building-in-Time. From Giotto to Alberti and Modern Oblivion* (New Haven and London: Yale University Press, 2010), 398.
54 GDSU, fols 34A, 35A, 36A.
55 GDSU, f.1636A, Porch of Maxentius; f.1638A, S. Sebastiano at Porta Capena.
56 GDSU, f.1637A, Temple of Fortuna Virile or S. Maria Egyptiaca at Porta Capena.
57 GDSU, f.4128A, Plan of the Basilica of Maxentius.
58 GDSU, f.1643A, Plan of S. Francesco a Ripa. Frommel attributed it to Peruzzi and dated it to 1509. See Christoph L. Frommel, '"Ala maniera e uso delj bonj antiquj": Baldassare Peruzzi e la sua quarantennale ricerca dell'antico', in *Baldassare Peruzzi 1481–1536*, eds Christoph L. Frommel *et al*. (Venice, Marsilio, 2005), 3–82 (21). For alternative attribution, see: Karen Perlove Shelley, *Bernini and the Idealization of Death. The Blessed Ludovica Albertoni and the Altieri Chapel* (University Park, PA, and London: The Pennsylvania State University Press, 1990), 55–56.
59 GDSU, f.465A.
60 Frommel, *op. cit*., 35–38.
61 Pietro Cattaneo, *I Primi Quattro Libri d'Architettura* (Venice: Figli di Aldo, 1554).
62 Philibert de l'Orme, *Le Premier tome de l'architecture* […] (Paris : Federic Morel, 1567), f.4v.
63 Florence, Biblioteca Nazionale, *Codex Magliabechianus* II.I.141, f.41r. See Martini, *op. cit.*, pl.233.
64 See Anthony Blunt, *Philibert de l'Orme* (1958), ed. Manuela Morresi (Milan: Electa, 1997), 183.
65 Galli Giovanni, 'A regulated suasion: the regulating lines of Francesco di Giorgio and Philibert de l'Orme', *Journal of the Warburg and Courtauld Institute*, 65 (2002), 95–131 (108).
66 Veronica Riavis, 'On a human scale. Drawing and proportion of the Vitruvian figure', *Diségno*, 7 (2020), 43–54 (50).
67 Cesare Cesariano, *Di Lucio Vitruvio Pollione de Architectura Libri Dece* […] (Como: Gottardo de Ponte, 1521), f.96v.
68 Sebastiano Serlio, *Tutte l'Opere d'Architettura et Prrospetiva* [i.e. *Prospettiva*] *di Sebastiano Serlio Bolognese* (Venice, Francesco de' Franceschi, 1600), fols 48v., 49r.
69 Bonaiuto Lorini, *Delle Fortificationi di Buonaiuto Lorini Libri Cinque*, I (Venice, 1597), 28–33. See Antonio Manno, 'Bonaiuto Lorini e la scienza delle fortificazioni', *Architettura. Storia e Documenti*, 2 (1985), 39–40. The Sienese engineer Teofilo Gallaccini would quote this technique in his unpublished *L'Idea della Fortificazione* per *Teofilo Gallaccini Matematico Senese. Ad Uso dell'Architettura Militare e dell'Arte della Guerra*, *c*.1630, Siena, Biblioteca degli Intronati, Ms.S.IV.3, c.69r.
70 Giovanni Fontana, *Bellicorum Instrumentorum Liber cum Figuris et Fictitiis Literis Conscriptus*. Cod. Icon. 2112, Bayerische Staatsbibliothek, Munich. See: Eugenio Battisti and Giuseppe Saccaro Battisti, *Le Macchine Cifrate di Giovanni Fontana* (Milan: Arcadia, 1984).
71 Francesco De Marchi, *Della Architettura Militare […] Libri Tre […]* (Brescia: Comino Presegni per Gaspare dall'Oglio, 1599). In any case, De Marchi still mentions Pliny's list of antique labyrinths as a model for modern fortified architecture. See Carpiceci and Colonnese, 'Labyrinth', *op. cit*.
72 Filarete explicitly listed the different widths of the labyrinthine moat of the Sforzinda castle. In a 1500br.-wide square, Filarete prescribed how to dig a trench. It must be 40 br. wide and 12 to 14 br. deep. Then he asked to repeat the operation six more times, sizing trenches and swathes of land according to different measures: 30 (trench) and 30 (land); 40 and 25; 40 and 25; 40 and 20; 40 and 20; and finally a 50 br. wide trench around the central square site. See Filarete, *op. cit*., 149. For his part, Serlio designed schemes in which the corridor gets narrower as it approaches the centre. See Serlio, *op. cit*., 199.
73 Fabio Colonnese, 'The labyrinth as an architectural mediator: Vredeman De Vries and the geometric garden in the Netherlands', in *Enchanted, Stereotyped, Civilized: Garden Narratives in Literature, Art and Film*, eds Sabine Planka and F. Cubukcu (Würzburg: Königshausen & Neumann Eds, 2018), 435–54.
74 Hans Vredeman de Vries, *Perspective c'est a dire, le très renommé art du poinct oculaire* (Lugduni Batavorum, 1604–1605), pl.28.
75 See those in the church of St Bertin at Saint-Omer and the Cathedral of Ghent. Hans Puechfeldner, who further developed Vredeman's results, presented labyrinthine gardens inspired by the labyrinths of Rheims Cathedral, St Bertein, and Ghent. See Hans Puechfeldner, *Nützliches Khünstbüech der Gartnereij,* 1590–1600. Rare RBR B Roller 3–3, Dumbarton Oaks Research Library, Washington DC.
76 See Sebastian Fitzner, *Architekturzeichnungen der Deutschen Renaissance: Funktion und Bildlichkeit Zeichnerischer Produktion, 1500–1650* (Germany: Modern Academic Publishing, 2015), 164–69, 574–76.
77 Daniel Loris, *Le Thrésor des parterres de l'univers: contenant les figures et pourtraits des plus beaux compartimens, cabanes, et labyrinthes des jardinages* (Geneva : Estienne Gamonet, 1629), 3–4.
78 Erik A. De Jong, 'For profit and ornament: the function and meaning of Dutch garden art in the period of William and Mary, 1650–1702', in *The Dutch Garden in the Seventeenth Century* (Dumbarton Oaks Colloquium on the History of Landscape Architecture, 12), ed. John Dixon Hunt (Washington DC: Dumbarton Oaks Research Library and Collection, 1990), 13–48 (22).
79 *Ibid*.
80 Konrad Ottenheym, 'Proportional design systems in seventeenth-century Holland', *Architectural Histories*, vol.2, no.1 (2014), 1–14 (1).
81 See Kenta Tokushige, *Pietro Cataneo and the Fortified City. The Use of the Grid Plan*, Masters Thesis (University of Virginia, 2016), 44.
82 Massimo Scolari, 'La Prospettiva Soldatesca', in *Il Disegno Obliquo. Una Storia dell'Antiprospettiva* (Venice: Marsilio, 2005), 264.
83 Massimo Scolari, 'Figure di dimostrazione', in *ibid.*, 202–28.
84 Hans Hubert, 'In der Werkstatt Filaretes', *op. cit.*
85 *Corpus Agrimensorum Romanorum*, Hyginus, *Constitutio Limitum*, C 136.18-22. MS Arcerianus A, 6th century CE. Wolfenbüttel, Herzog August Bibliothek: Cod. Guelf. 36.23 Aug.
86 Hans Hubert, 'Freihand-, Raster-, Schauzeichnung. Bramantes Entwurfsmethodik', paper presented on 4 October 2014 at the International Congress 'Inventor e luce della buona e vera Architettura': *Bramante e gli 'Ordini Nuovi' nell'Architettura del Cinquecento e Oltre* (2–4 October 2014, Accademia di S. Luca, Rome): https://www.youtube.com/watch?v=zq4HtOmbj-0 [accessed 17.10.23].
87 Wolfgang Lotz, 'Sull'unità di misura nei disegni di architettura del Cinquecento', in *L'Architettura del Rinascimento*, ed. Massimo Bulgarelli (Milan: Electa, 1997), 213–19 (217) (trans. by the author).
88 Pérez-Gómez, *op. cit*., 19.
89 A Dutch edition of Vitruvius's treatise was planned also by Jacob van Campen and Constantijn Huygens, father of the mathematician Christiaan, who had translated the Latin treatise together with Count Johan Maurits van Nassau-Siegen. See Ottenheym, *op. cit*, 12.
90 Claude Perrault, *Les Dix Livres d'Architecture de Vitruve Corrigez et traduitz nouvellement en françois avec des notes et des figures* (Paris: Jean Baptiste Coignard, 1673).
91 Pérez-Gómez, *op. cit*., 109.