The Sun as Drawing Machine: Towards the Unification of Projection Systems from Villalpando to Farish —

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The difference between the appearance of a body for us and for God is the difference between scenography and ichnography.

Leibniz, 'Letter to Des Bosses' (1712), quoted in Jonathan Crary, *Techniques of the Observer: On Vision and Modernity in the Nineteenth Century* (Cambridge, MA, and London: MIT Press, 1990), 51.

As for the uses of shadow, besides that it serves to avoid the heat of the Sun, & its inconveniences, it represents all kinds of bodies, & seems to have given birth to painting, and to all the arts which teach the method of representing something.

Jean François Niceron, *La Perspective curieuse* (Paris: Jean Depuis, 1663), 48–9.

'Virtual machines' and representation

To what do we refer when we use the term 'drawing instruments'? Certainly, material objects like set-squares, rulers and pantographs, devised to enable drawing operations. Yet we can also consider as instruments those 'virtual machines' that, without the mediation of our dexterity, serve to capture a non-subjective image of a three-dimensional body.[1] Examples are to do with gravity or light projection, and the material devices that come to be based on them, such as the plumb line and the camera obscura. In western culture, such instruments have guaranteed the 'truth' of a drawing, at certain times endowing the forms of representation they generate with a higher ontological status.

An early and influential architectural example is found in Vitruvius. The plan (*ichnographia*) of a building is related to gravity, which transfers its imaginary footprint to the ground, while the elevation (*orthographia*) remains as a mere procedure of translation of measures.[2] As each of the types of drawing Vitruvius considers (plan, perspective and elevation) is linked to a different 'virtual machine', so they are conceptually separated from one another – and this is the situation that the Renaissance will inherit. To further complicate the situation, during the 17th and 18th centuries forms of representation unauthorised by Vitruvius emerged – the 'proto-axonometric' drawings,[3] which, not being the result of any 'virtual machine', lacked objective legitimacy and tended to be used only for the representation of specific domains of reality.

'Virtual solar machines', transparent shadows, and the concept of orthogonal projection

The hypothesis of this article is that the conceptual development of a 'virtual solar machine' during the 17th and 18th centuries would have played a significant role in the construction of the concept of projection

Fig.1 The sun and the candle's shadows and the origin of painting. From Joachim von Sandrart, *L'Academia Todesca della Architettura, Scultura e pittura: Oder Teutsche Academie der Edlen Bau- Bild- und Mahlerey-Künste*, Vol. 1, 2 (Nuremberg, 1675), plate II. Public domain, courtesy Deutsches Textarchiv.

on which the creation of modern systems of representation starting in the 19th century is based. Throughout the Renaissance, while Vitruvius' text was 'recovered' and discussed, narratives circulated inherited from the Greco-Roman world concerning the delineation of shadows. As is well known, Pliny's story of Butades' daughter tracing the shadow of a young man on a wall was taken to represent the 'origin of painting' (whether it was in lamp- or sunlight is unclear). In another story, Quintilian relates it to the shadow that a shepherd traces with his staff on sand (there is no doubt that this is a shadow cast by the sun). Studies have shown how these stories were collected (by Vasari and Alberti, among others) and spread through engraving and painting, with the tale of Butades enjoying extraordinary popularity at the end of the 18th century.[4] An illustration from Joachim von Sandrart (1675) can serve as a visual registration of these two narratives (Fig.1).

What I wish to emphasise here is that these two narratives suggested the possibility of creating two drawing machines – one using a light-source such as a candle or torch, and the other the light of the sun. While examples of the 'candlelight machine' and its association with perspective have been studied and are well known, the development of the 'solar machine' has gone largely unremarked upon. It is possible to surmise that the conjunction of both luminous 'machines' might have stimulated a first synthesis of what today we call 'projection systems' (whether parallel or radial).[5] In this article, I have collected material to verify this hypothesis, trying to locate these machines or virtual instruments that 'draw' using shadows projected by the sun and to understand their role. The period covered stretches from the latter part of the 16th to the early 19th century, when isometry was defined. As we will see, this 'virtual solar machine' becomes possible thanks to the development of the concept of a transparent body and its corollary, its transparent shadow. Therefore, it is worth saying something about its origin.

As Victor Stoichita has pointed out, in the fictional accounts of Pliny and Quintilian, the shadow is only a silhouette, and thereby ignores everything inside the outline – a fact that significantly limits the possibility of these stories inspiring 'effective drawing machines' and inevitably implies that reflection is a superior model.[6] Stoichita, however, does not mention how an intermediate way emerged, a surprising product of our Western graphic culture – the invention of the 'transparent shadow', which is linked to the emergence of the also unique concept of *corpo transparente* as opposed to *corpo solido*.

Corpo transparente and transparent shadow

The idea of representing bodies in these two ways seems to begin with Renaissance studies of Platonic solids. They appear in Leonardo da Vinci's drawings for Luca Pacioli's *Compendio de Divina Proportione* (1498), labelled as *planus* and *vacuus*. It is a representation favouring the assimilation by the spectator of the geometrical properties of the bodies, and pointing to a vision of the world that, following Platonic and Pythagorean doctrines, understands that behind the appearance of sensually apprehended reality, there exists a deep form which the drawing could bring to light.[7] However, what was initially limited to Platonic solids would be extended by Sebastiano Serlio (1475–1554) to the representation of any material body that we want to draw. In his second book on the construction of perspective (1551), Serlio coins the terms 'transparent body' and 'solid body'. The draughtsman will make the transparent version first (in his example, an octagonal prism [Fig.2]); and this will benefit him in a way comparable to that in which a knowledge of anatomy benefits those depicting living creatures.[8]

The notion of the 'transparent body' will extend throughout the period we are studying here, from the field of painting to that of architectural representation, and will eventually give rise to a novel analytical drawing.[9] In 1620 Bernardino Amico of Gallipoli published a remarkable wire-like diagram of the interior space of the Church of the Sepulchre of the Blessed Virgin in the Holy Land, which he described as 'a transparent body [*corpo transparente*] ... which, by means of its visual lines, shows in perspective the space enclosed by a building devoid of walls and enclosures' (Fig.3).[10] While traditionally a body was understood as opaque and cast opaque shadows, in its version as a *corpo transparente* (drawn or materialised in 'wireframe' or hollow models) it would cast 'transparent shadows', within which previously hidden features would be seen to delineate themselves. It was enough to place this transparent body under a light source – the sun – to conceive a possible drawing instrument. As we will see, this instrument, composed of a sun illuminating a transparent model, usually has a virtual character.

From gravity to sunlight: Vitruvian *ichnographia* as transparent cast shadow

We find a striking first example of this virtual instrument in a work of the Spanish Jesuit Juan Bautista Villalpando (1552–1608) devoted to elucidating the Temple of Solomon. Villalpando had been sent to Rome in 1592, accompanying another priest, Jerónimo de Prado (1547–1595), with the mission of thoroughly interpreting the Book of Ezekiel, which contained a vision of a sacred building that both Jesuits identified with the original Temple. Villalpando – who apparently had collaborated with Juan de Herrera in the design of El Escorial – would oversee the chapters related to the architectural description of the edifice. The endeavour would consume the rest of their lives – three years after arriving in Rome, Prado died, and Villalpando had to continue alone until his own death in 1608. Fortunately, thanks to the financing of King Philip II, the enormous effort would not be in vain, and the work was published in three splendidly illustrated volumes entitled *In Ezechielen Explanationes et Apparatus Urbis ac Templi Hierosolomitany* (1595–1606) (Fig.4).

Fig.2 Serlio praises the profound perception of form that may be acquired by visualising a version of a 'transparent body' from a 'solid body'. From Sebastiano Serlio, *Il Primo (-secondo) Libro d'Architettura* (Venice, 1545), 35–36. Public domain, via Internet Archive/Getty Research Institute.

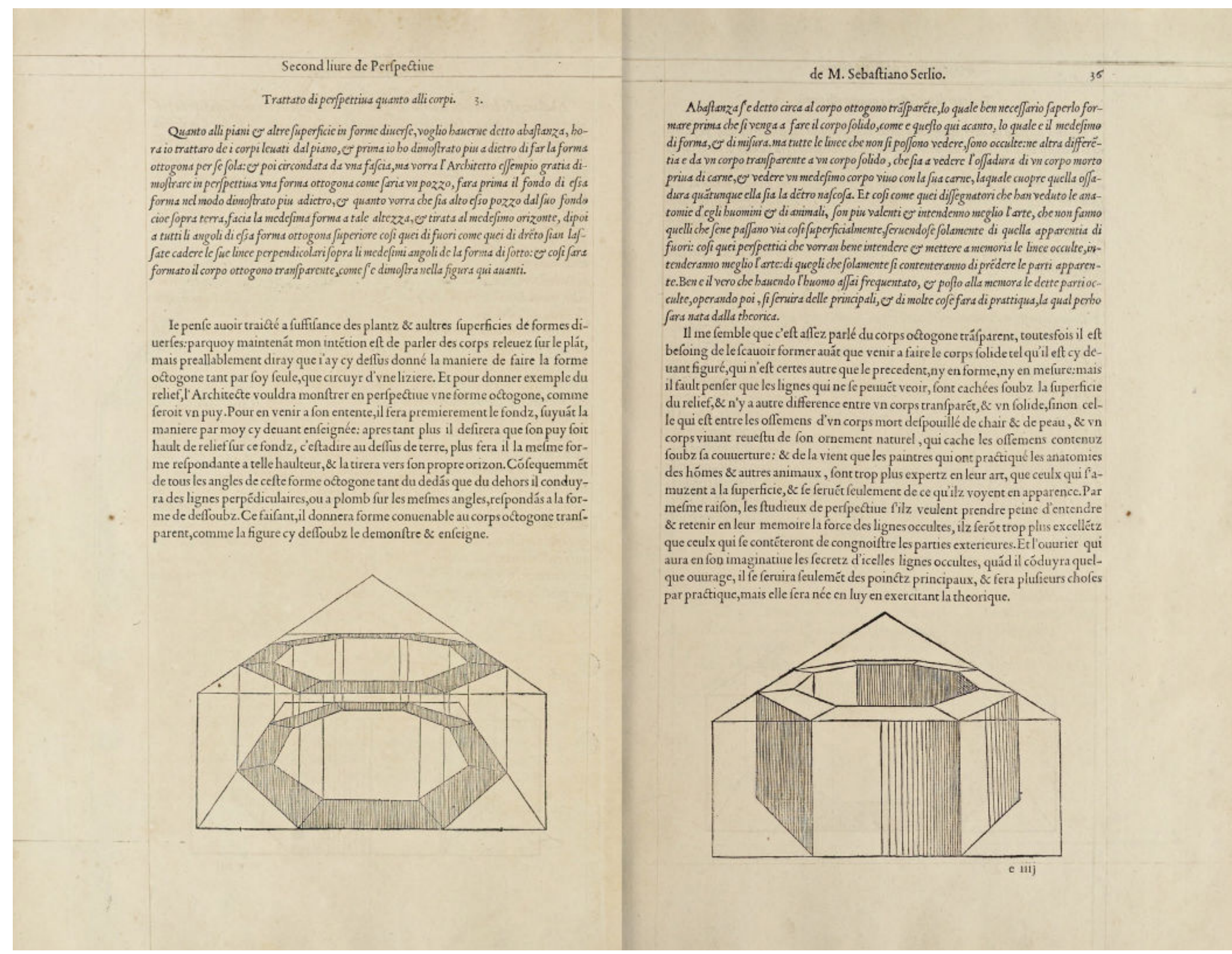
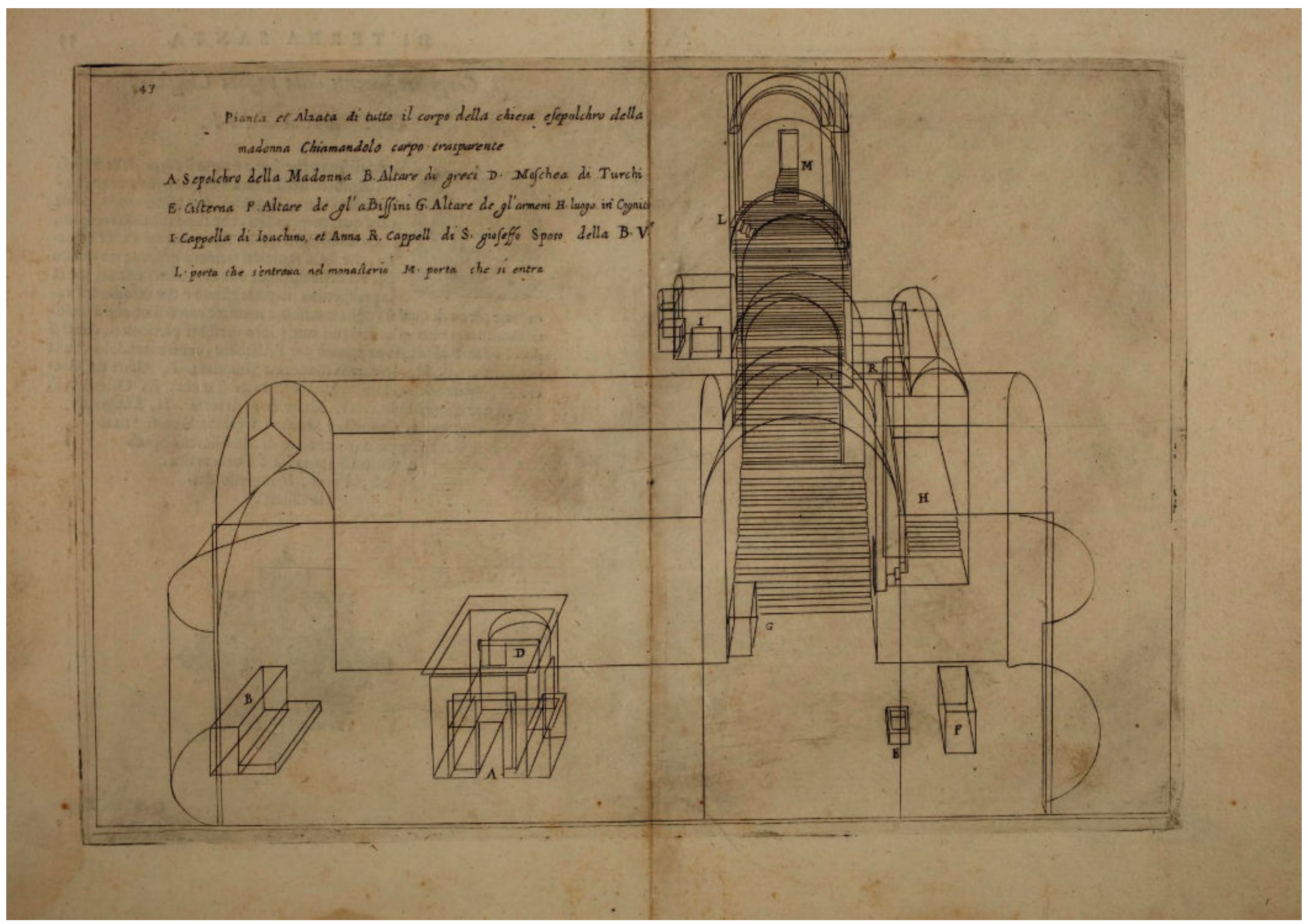


Fig.3 (Jacques Callot) Plate 43, 'Pianta et Alzata di tutto il corpo della chiesa esepolchro della Madonna Chiamandolo corpo transparente', in Bernardino Amico, *Trattato delle Piante & Immagini de Sacri Edifizi di Terra Santa Disegnate in Ierusalem secondo le regole della Prospettiua, & uera misura della lor grandezza* (Florence: Pietro Cecconcelli, 1620). Public domain, via Internet Archive/Sterling and Francine Clark Art Institute.

Fig.4 Frontispiece of *De postrema Ezechielis Prophetae visione*, second volume of Juan Bautista Villalpando and Jerónimo del Prado, *In Ezechielen Explanationes et Apparatus Urbis ac Templi Hierosolomitany* (Rome, 1605). Public domain, courtesy Biblioteca de la Universidad de Sevilla.



In the second volume, *De postrema Ezechielis Prophetae visione*, Villalpando tries to demonstrate that the Temple, which had been directly inspired by God, also met the Vitruvian principles of harmony (Fig.5). Throughout his argumentation, Villalpando demonstrates knowledge of the most advanced science of the time across areas such as music, mathematics, and – especially – mechanics.[11] To see how Villalpando proposes his 'solar machine', we will turn to an explanation in Chapter 12 of this volume, 'What does ichnography offer architects, and how is it done?'[12] In this, Villalpando first defines what Vitruvius means by *vestigium*: it is an impression 'absolutely similar and equal to the sole of the foot'. According to him, this Latin term would be translated into Greek by *ichnographia*, into Spanish and Italian by *planta*, and into French, following 'Philandro' (Guillaume Philander), as *quasi plana*.[13] But Villalpando then introduces a radical change in his argument – the plan would not be the outcome of a 'machine' that presses or acts under gravity, but a result of optics. This is a shift that would allow the unification of all graphic documents describing architecture under the same source of legitimisation. As he emphasises: 'we have reduced ichnography ... to the precepts of optics, on whose principles it is founded, no less than orthography or scenography itself' (Figs 6a, 6b, 6c).[14] Defining *ichnographia* as 'a graphic description ... that describes the plan of a reduced building [a model] that is seen by an eye equal to the same building', he then goes on to imagine that, instead of the eye, it is light which draws its plan.[15] Thus:

> If we place the model of a building on a board or on the flat pavement and we illuminate it with a light source equal to the same building, superimposed and perpendicular to it, the shadow, certainly parallel, will make noticeable on the pavement all the limits of the building. If you remove the building, make these limits remain, and somehow separate the illuminated parts of the pavement from the shaded parts, you would describe the ichnography of the building on the pavement.[16]

And if we were to imagine the building as a transparent body, this shadow would disclose all the information relating to it in a single drawing. As Villalpando writes: 'if the whole building were diaphanous, all its limits would be seen, the intermediate parts not impeding its vision, from which it follows that the intelligent architect knows in ichnography the disposition, magnitude, and place of each of the parts'.[17]

Villalpando concedes that in practice this could engender 'no little confusion', and that is why architects are in the habit, when drawing a building with several floors, of making 'an ichnography for each of these'. Even so, based on this imaginary 'solar machine', he proposes a new form of representation akin to what, in digital representation, we will come to understand by 'layers':

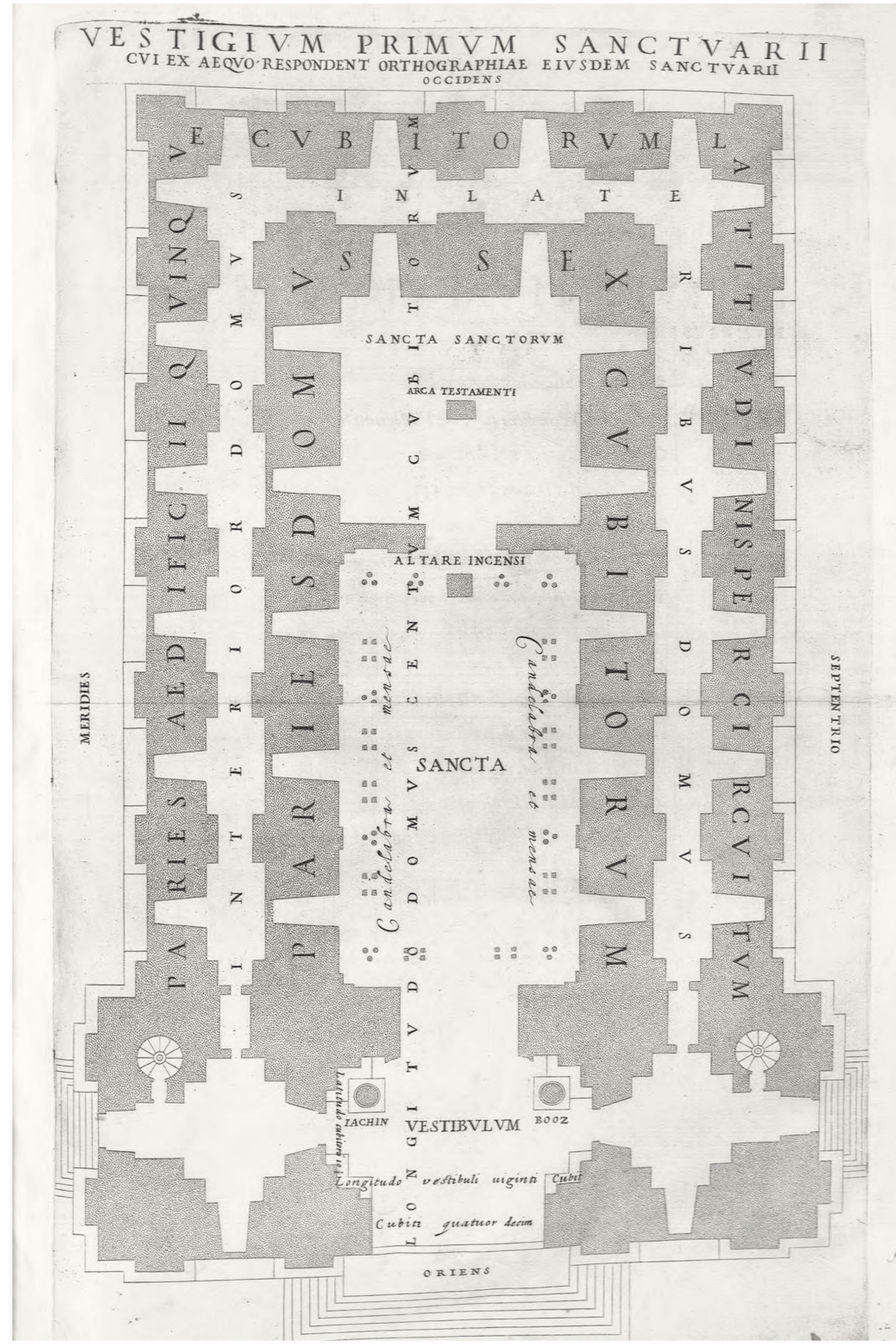
> We have judged it opportune to offer before a piece of salubrious advice to the architects, namely, that they describe in one ichnographia all the levels, and then distinguish with lines of different colours the different plans of the floors, and then transfer them separately to various papers. This way, all the superior members will correspond adequately to the inferior ones.[18]

The 'solar machine' of Villalpando will reappear 70 years later in the work of the Cistercian monk and polymath of Spanish origin, Juan Caramuel Lobkowitz (1606–1682). In his *De la Arquitectura Civil* (1678–79) – his only work devoted to architecture – he fiercely criticises the errors of interpretation of Villalpando. However, this does not prevent him from agreeing that if the sun at its zenith illuminates a transparent version of the building (*omnes lapides in plano adumbret*), the shadow thus produced would coincide with the '[i]chnographia, which outlines the whole building on the plane'.[19] This is why, according to Caramuel, the hidden lines of projection in one of his illustrations should be seen as the 'shadows of various mouldings'.[20]

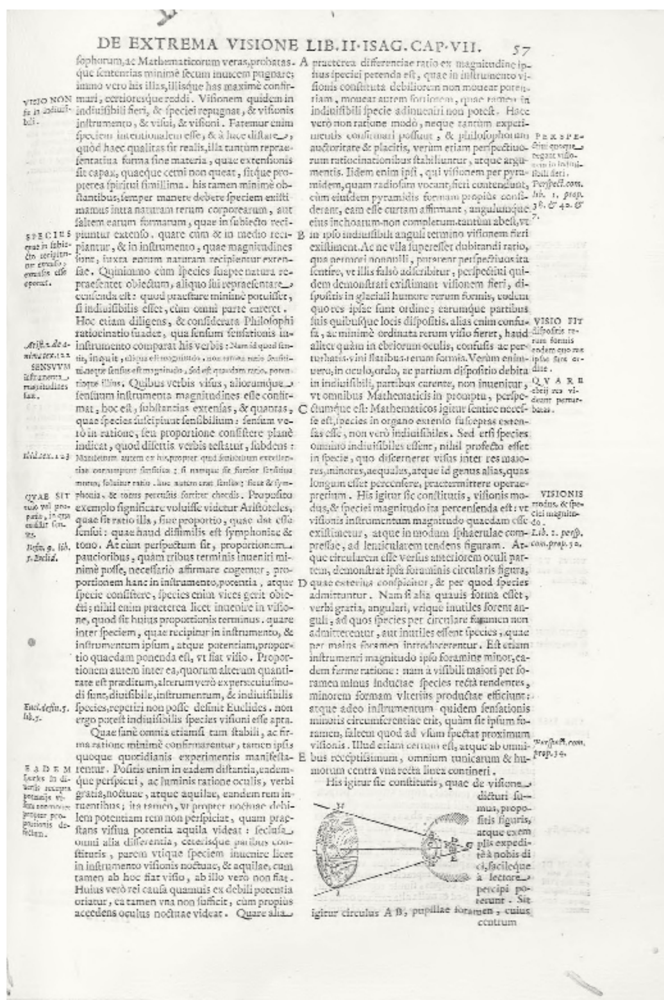
The Catholic thought of the Counter-Reformation – particularly that of the Jesuits – had established a link between divine emanation and the sun. Caramuel, elsewhere in his work, explicitly addresses how 'the sun signifies Christ'.[21] It is a connection worth exploring in relation to the two authors, and might offer a clue as to why both resorted to the idea of the 'transparent body'. According to Alberto Pérez-Gómez and Louise Pelletier, Villalpando had in mind 'an infinite God, identified with the light of the sun, omnipresent in the world of experience and yet casting "precise" parallel shadows'.[22] However, symbolic localisation was not easy given the entanglements of Catholic theology. Robin Evans has discussed the complications that faced Renaissance painters and architects who tried to organise the centres of attention of their works to correspond with the existence of a God who simultaneously envelops creation and is the focus from which it emanates.[23]

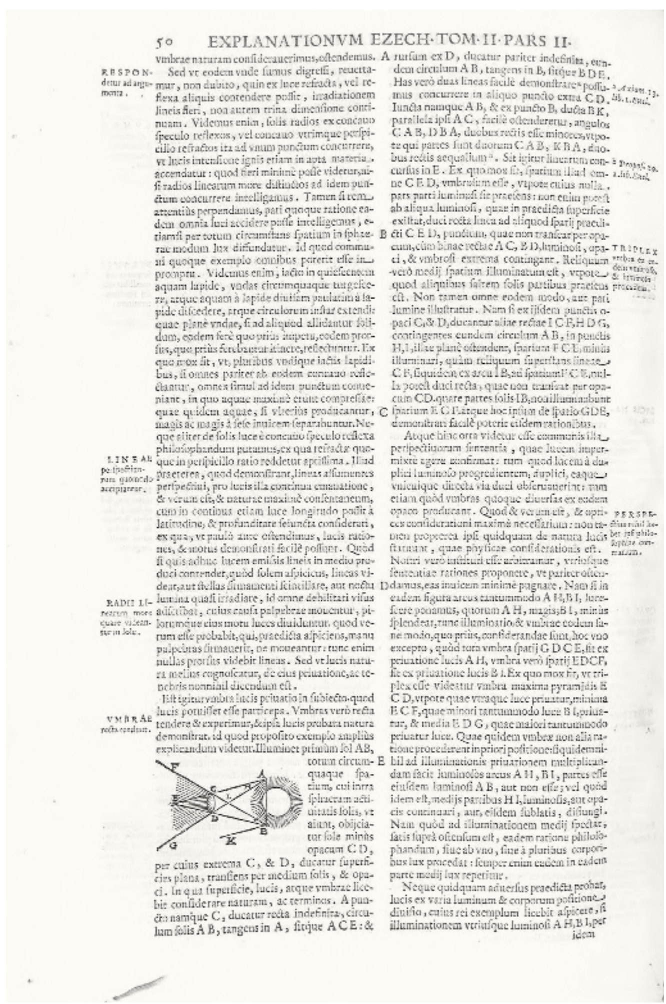
In my view, Villalpando's machine would not escape – if we take the symbolic divinisation of the sun seriously – the type of conflict that we could call 'topological'. An omniscient God identified with the sun must also be able to see shadows, and Villalpando's imaginary of a diaphanous building – as an object made transparent by its subjection to the divine gaze – might be a way of resolving this. However, perhaps less curious than the fact that both conceived buildings as transparent bodies (we have already noted how in 1609, only a year after Villalpando's death, Bernardino de Gallipoli published his extraordinary *corpo transparente* drawings of the interior of a building) was the very invention of the 'solar machine' itself – that is, the placing of a body under the sunlight to generate an image from its shadow. To what extent was this a novel idea?

Fig.5 'Vestigium Primum Sanctuarii'. *Ichnographia* (plan) of the Temple, Plate I of *De postrema Ezechielis Prophetae visione*, second volume of Juan Bautista Villalpando and Jerónimo del Prado, *In Ezechielen Explanationes et Apparatus Urbis ac Templi Hierosolomitany* (Rome, 1605). Public domain, courtesy Biblioteca de la Universidad de Sevilla.



Figs 6a, 6b, 6c *Ichnographia* as transparent shadow in Villalpando's *De postrema Ezechielis Prophetae visione*. Schematic drawings supporting his discussion on light and optics in chap. 4, lib. II. From Villalpando and del Prado, *In Ezechielen Explanationes et Apparatus Urbis ac Templi Hierosolomitany* (Rome, 1605). Public domain, courtesy Biblioteca de la Universidad de Sevilla.





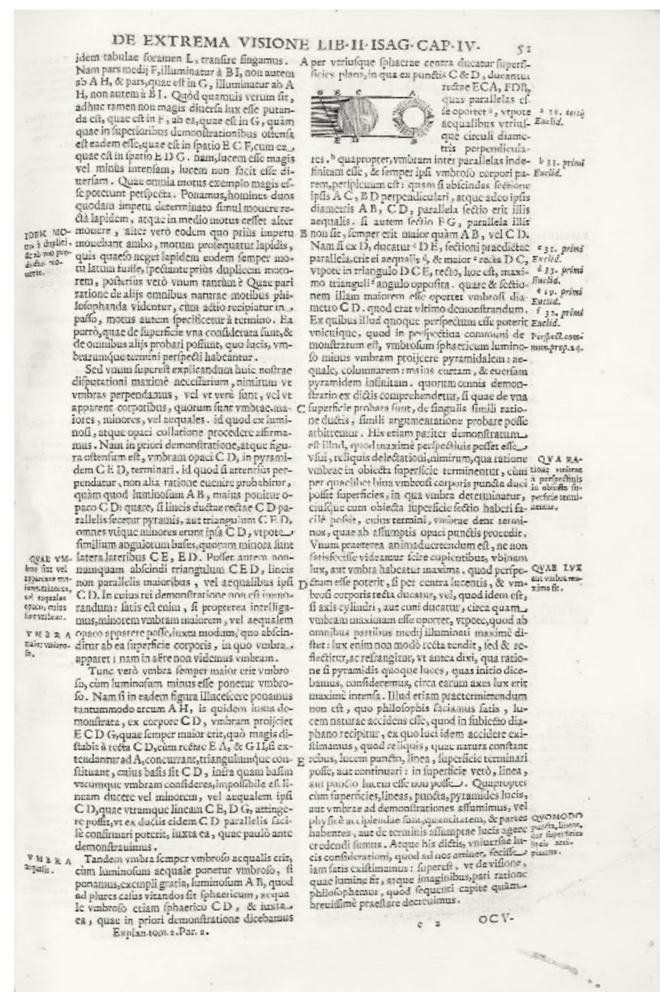
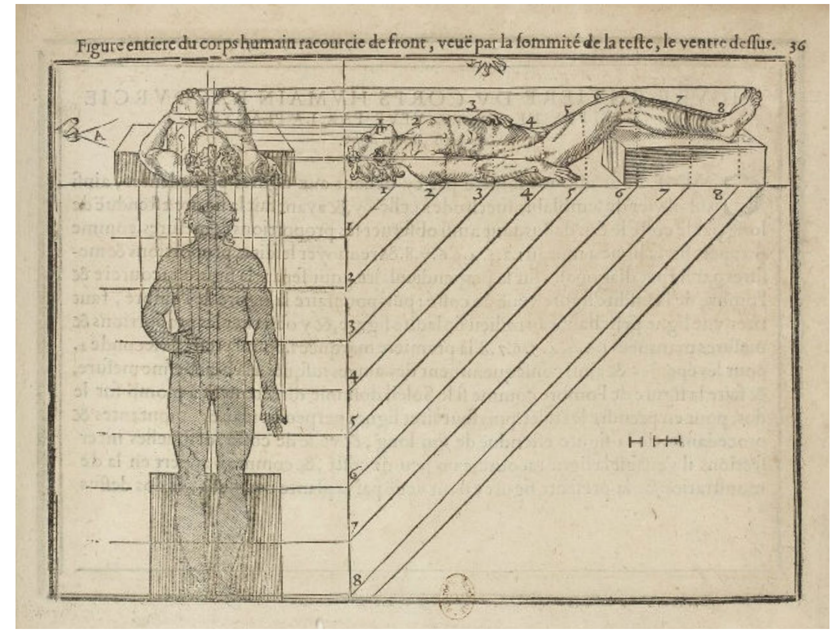


Fig.7 Sunlight and transparent shadows in Cousin's method of foreshortening the human body. From Jean Cousin, *La Vraye science de la Pourtraicture et demonstrée par Maistre Jean Cousin, peintre & geometrien* (Paris: chez Guillaume Le Bé, 1656), 36. Public domain, courtesy Bibliothèque nationale de France, dép. Estampes et photographie, 4-KC-2 (B).

Fig.8 Andrea Mantegna, *Lamenatation over the Dead Christ*, *c.*1483. Tempera on canvas, 68 x 8cm. ©Pinacoteca di Brera, Milan.



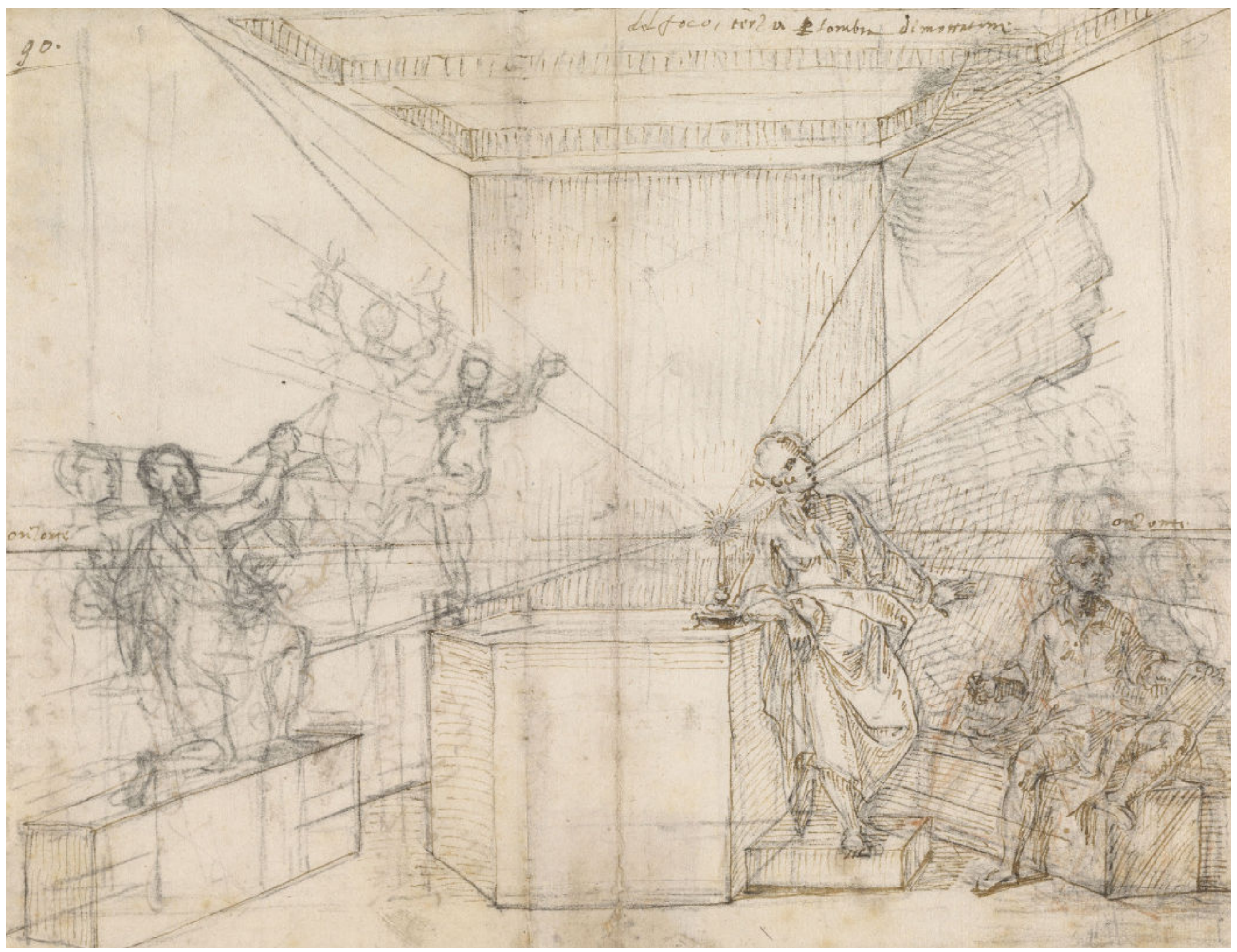
The sun as a device for drawing human bodies: the case of Jean Cousin

To answer this, we must explore other fields of graphic representation. Caramuel had recalled how the main challenge for painting since its mythical birth, as described by Quintilian, had been to know how to complete the interior of the silhouette projected by the sun. Might the 'solar machine' that projects transparent shadows have already appeared in pictorial theory as a possible solution?[24]

Let's turn to an earlier case applied to the pictorial representation of the human body. In 1571 Jean Cousin (the younger, 1522–1594) published a book that would have enormous influence, *La Vraie science de la portraicture*, going through a considerable number of editions (under a somewhat different title from 1663).[25] In this, Cousin deals with a particular problem of representation – the foreshortening of the human body. One of the most striking aspects is that the procedure he follows produces the figure's shortening through an oblique projection. This may seem bizarre, since it may imply that two rules can coexist in the same painting, with elements of architectural scenography foreshortened according to the rules of perspective and human bodies reduced orthographically. However, authoritative commentators such as Jacques-Nicolas Paillot de Montabert (1771–1849) would later praise Cousin's approach,[26] arguing that he was not alone, his approach being anticipated by painters such as Andrea Mantegna, among others. (It is suggestive to compare Cousin's plate 'Figure entiere du corps humain racourcie de front, veuë per la sommité de la teste, le ventre dessus' [Fig.7] with Mantegna's *The Lamentation over the Dead Christ* [*c.*1483; Fig.8]. In this, the bed is diminished while the body seems to be shortened in orthogonal projection, thus preserving the full dignity of the head of the recumbent Christ, in comparison with his feet in the foreground). Even more surprising is that the graphic process for the foreshortening of the human body involves obtaining a 'transparent' shadow cast by a virtual sun, as Cousin often explicitly points out in the text that accompanies his plates. Why did he resort to this?

Painters could see in Cousin's method an alternative to the use of lamps advocated in certain schools for the foreshortening of the human body (as illustrated by the well-known image in the Huygens Codex [Fig.9]), since the projected shadow produces deformations that are difficult to master via perspectival technique. In other words, more educated artists would see that Cousin's procedure connects with the parallel (and not radial) rays of Quintilian's account, and is preferable for its simplicity and formal constancy to shadow projection via a lamp, which connects with Pliny's account, with the additional advantage of indicating how to draw the interior of the shadow.

Fig.9 Drawing a candlelit human body's shadows on the wall. Carlo Urbino, *Del foco. Terza per l'ombra dimostratione*, Codex Huygens, f.90, *c*.1560–70. Black chalk, pen and brown ink, red chalk, lines inscribed with stylus on laid paper. 18.2 x 23cm. Morgan Library & Museum, New York (2006:14).



Now, what kind of perspective is this? Cousin risked being misinterpreted. Grégoire Huret's scathing critique (1670) was based on what he saw as a tremendous mistake. According to Huret, the bodies drawn by Cousin:

> do not admit any point of view or position of the eye ... [establish] infinite points of view for a single figure, & consequently infinite positions of the eyes of the beholder, who should even be all covered with eyes to see it well, or rather have each of his eyes as large as the whole picture.[27]

Cousin's sun, however, would precisely respond to this interpretation by refuting such a totalising optical condition. In his plates, Cousin carefully distinguishes the sun that produces the shadow from the human eye that contemplates it from a nearby position. There is no such generalised spectator. The sun is not an eye as big as the object, as Villalpando will describe it; instead, it is simply a focus.

Transparent shadows and 'proto-axonometry': Pietro Accolti and the blindness of the sun

If, in Villalpando, the 'virtual solar machine' justified the plan of a building, and, in Cousin, a parallel projection of a body, would those transparent shadows generated by the sun give theoretical support to the 'proto-axonometries' that proliferated throughout the 17th and 18th centuries? As we shall see below, there were times in the 17th century when such recognition seemed imminent – yet ultimately there was a failure to recognise that the shadows they drew for other purposes coincided with the 'proto-axonometric' images of 'military views'.

The first case we will discuss is a striking illustration in Pietro Accolti's (1579–1642) *Lo Inganno de gl'Occhi*, published in 1625. In this treatise, in which he deals with a specific problem of perspective (the drawing of the shadows cast by bodies), we find an image of a cube (with an octagon inscribed in each face, which he termed *organo ombrifero*) and its transparent shadow that reminds us of what today we would call 'military axonometry'.[28] Where does this image come from? Accolti wanted to solve the problem of putting into perspective the shadows cast by the sun on a regular body. To do this, he proposed a two-step procedure. First, the sun would draw the transparent shadow of the model on a horizontal surface in its true magnitude; then the draughtsman would copy and manipulate it according to the rules of perspective (Fig.10). One wonders whether in this case we are dealing with a 'virtual machine' or a real one. Certainly, it seems more like a mental experiment – otherwise we are faced with the laborious creation of a model for each object.

As with Villalpando, Accolti clearly and explicitly identifies the sun as an 'eye that sees' with parallel rays, drawing from this an even more surprising conclusion – the sun is an eye condemned never to see the shadows it casts.

> Therefore ... we understand that the Sun never sees any shadow of the opaque surfaces, which he contemplates and illuminates, so we intend all that comes into his sight to remain illuminated, while on the contrary, all that is hidden to remain shadowy and deprived of his splendour.[29]

Ironically, the human being has a power that the sun lacks, able to perceive what it can never see – an extraordinary observation to which Filippo Camerota has drawn attention because of its possible relationship with Galileo's discoveries.[30]

The truth is that this was an idea already in circulation, so it could have shaped how both Accolti and Galileo thought about shadows.[31] But beyond the vertigo provoked by these conclusions, this 'sun able to see' will have a decisive impact on the future development of the concept of axonometry. As we shall see, in an exchange of roles, its humanisation will make it possible to substitute the spectator in infinity for the sun.

Transparent shadows and 'proto-axonometry': the *ichnographia* of floating bodies in the treatise of Jean Dubreuil

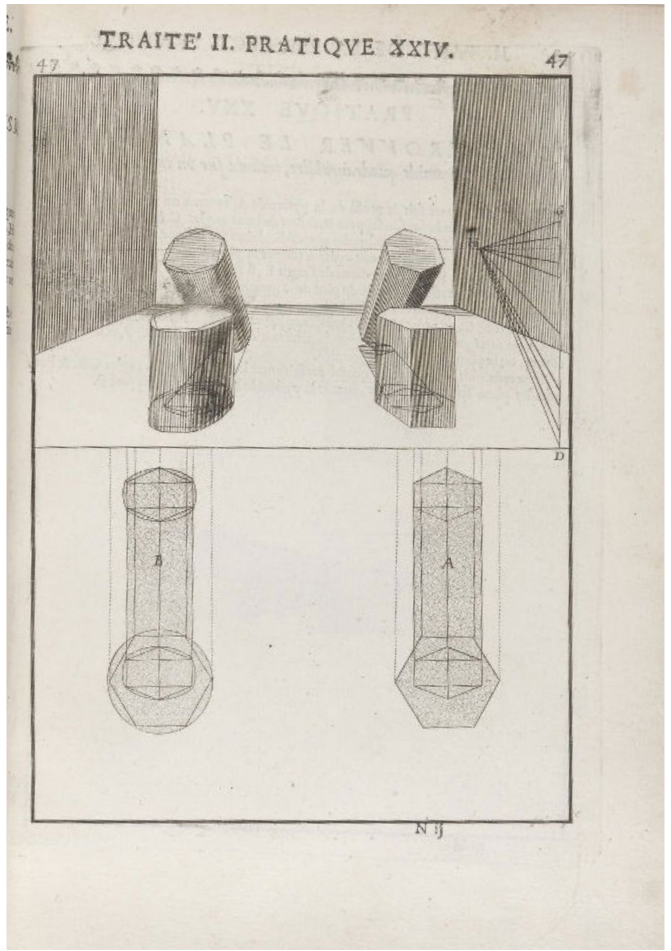
There is a further work on perspective in which we again encounter transparent shadows virtually cast by the sun, whose images suggest an axonometry of the bodies that project them: Jean Dubreuil's *Perspective pratique* (1642–1649, 1679). In the third volume of his treatise, Dubreuil (1602–1679) states that the first step in solving the problem of constructing the perspective of various geometric bodies in different positions in space is to have an objective description of them. This is obtained by using a projection of parallel beams at right angles to the horizontal plane upon which the bodies 'float and rotate'.

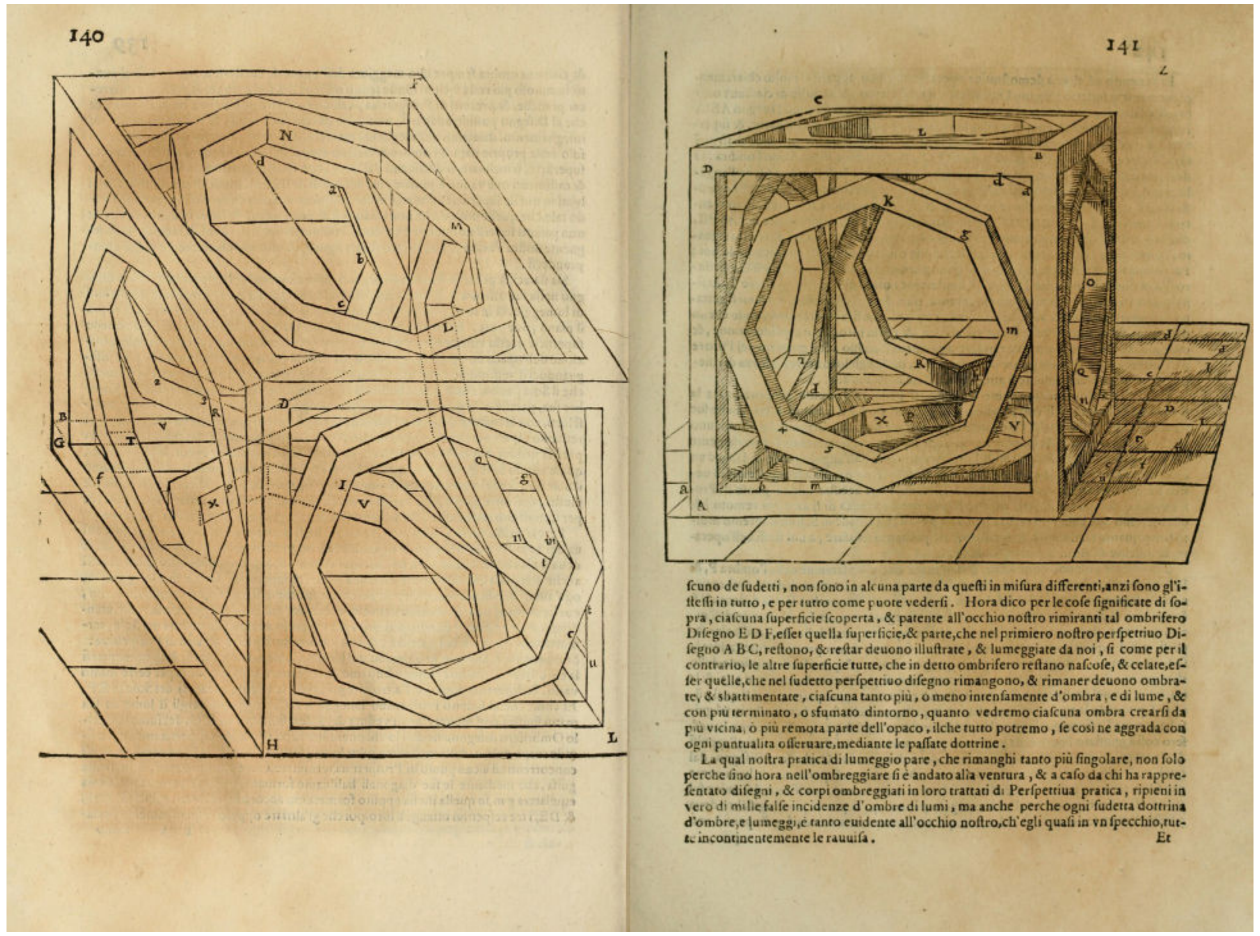
For Dubreuil, the images thus obtained could be seen as the transparent shadows cast by these bodies under the sun at its zenith. He called them *ichnographies* which broadened a concept hitherto limited to the traces of a body resting on a horizontal plane.

> I call the shadows that render bodies illuminated by the Sun when it is directly above, the Ichnographic Plan, which is the correct name for what is commonly called Geometral. This I have not done without reason. Because if for the purely Geometral or Ichnographic plan, we mean a trace that represents the vestiges that would be on earth, the foundations of that which we want to raise; this name in the art of Perspective is not poorly suited to the shadows that solid bodies make when the Sun falls on them perpendicularly.... This is why when we say, Ichnographic plan, one must understand the shadow of these bodies illuminated by the Sun: & by the Perspectival plan, the same Ichnographic plan put in Perspective.[32]

Fig.10 A ‘proto-axonometric’ image. On the left, the *ombrifero*, a sunlit transparent shadow of a body in Pietro Accolti’s *Lo Iganngio*, which is meant to be put, after that, as shown on the right, into perspective. From Pietro Accolti, *Lo inganno de gl’occhi, prospettiva pratica*, vol. 2 (Florence: Appresso Pietro Cecconcelli, 1625), 140–141. Public domain, via Internet Archive/Getty Research Institute.

Figs 11a, 11b, 11c ‘Proto-axonometric’ and ‘proto-isometric’ images in Dubreuil’s *Perspective pratique: the ichnographie* as the transparent shadow and first step in constructing the perspective of regular bodies. From Jean Dubreuil, *La Perspective pratique, necessaire a tous peintres, graveurs, sculpteurs, architectes, orphevres, brodeurs, tapissiers, & amp; autres qui se meslent de desseigner* (Paris: Antoine Dezallier, 1679). Public domain, courtesy Bibliothèque nationale de France.

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ſcuno de ſudetti, non ſono in alcuna parte da queſti in miſura differenti,anzi ſono gl'iſteſſi in tutto, e per tutto come puote vederſi. Hora dico per le coſe ſignificate di ſopra, ciaſcuna ſuperficie ſcoperta, & patente all'occhio noſtro rimiranti tal ombrifero Diſegno E D F,eſſer quella ſuperficie,& parte,che nel primiero noſtro perſpettiuo Diſegno A B C, reſtono, & reſtar deuono illuſtrate, & lumeggiate da noi, ſi come per il contrario, le altre ſuperficie tutte, che in detto ombrifero reſtano naſcoſe, & celate,eſſer quelle,che nel ſudetto perſpettiuo diſegno rimangono, & rimaner deuono ombrate, & sbattimentate, ciaſcuna tanto più, o meno intenſamente d'ombra, e di lume, & con più terminato, o sfumato dintorno, quanto vedremo ciaſcuna ombra crearſi da più vicina, ò più remota parte dell'opaco, ilche tutto potremo, ſe così ne aggrada con ogni puntualita oſſeruare,mediante le paſſate dottrine.

La qual noſtra pratica di lumeggio pare, che rimanghi tanto più ſingolare, non ſolo perche ſino hora nell'ombreggiare ſi è andato alla ventura, & a caſo da chi ha rappreſentato diſegni, & corpi ombreggiati in loro trattati di Perſpettiua pratica, ripieni in vero di mille falſe incidenze d'ombre di lumi, ma anche perche ogni ſudetta dottrina d'ombre,e lumeggi,è tanto euidente all'occhio noſtro,ch'egli quaſi in vn ſpecchio,tutte incontinentemente le rauuiſa.

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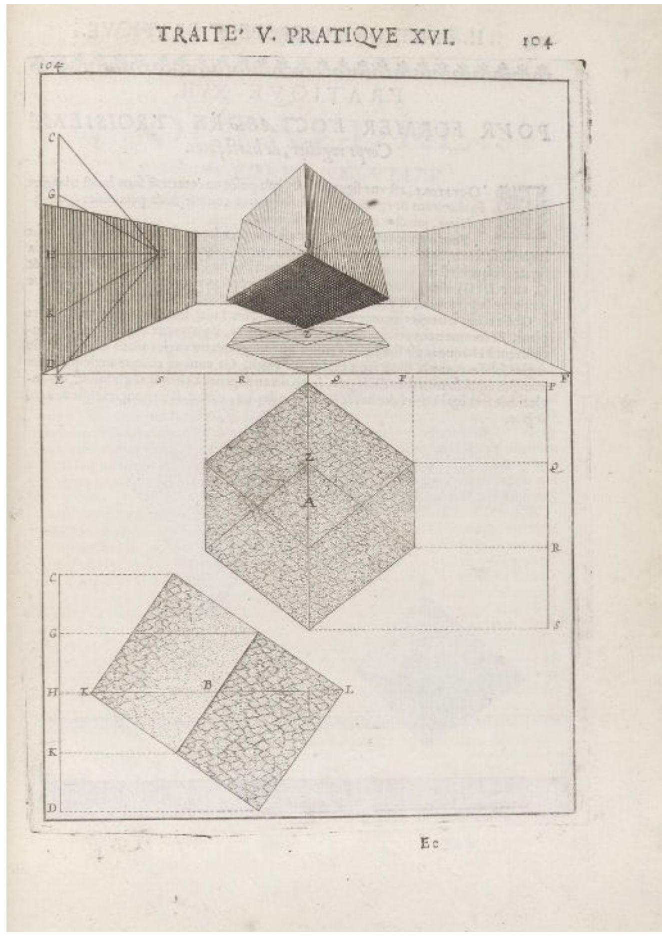
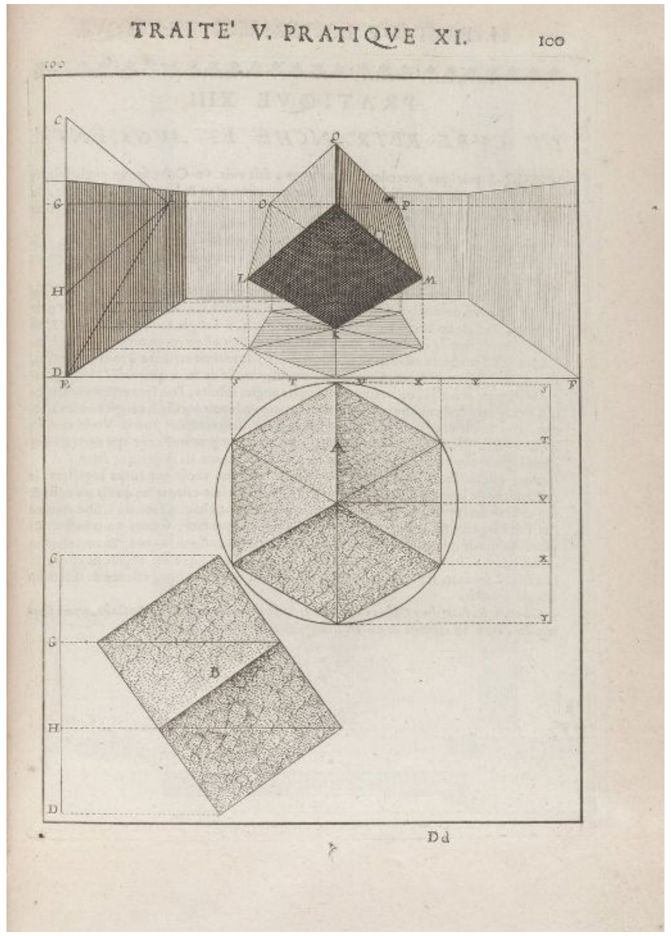




Throughout the treatise, there are many of these constructions. We see regular bodies in multiple positions in space that are projected orthogonally and which – he often insists in the text – are sorts of 'transparent shadows' drawn by the sun. This is the case with, for example, the isometric-like projection of a cube shown in 'Traité V, Pratique XI', 'finding the plane of a cube raised on an angle' (Fig.11c).[33] As before with Accolti, we ask: does Dubreuil sense that these shadows may be the basis for building an 'axonometric' projection system? Again, this does not seem to be the case. Faced with the 'isometric' shadow of the cube he limits himself to pointing out that 'its shadow … in this situation gives a perfect & circular hexagon', that is, a flat figure.[34] He does not see the similarity between his shadows and a hypothetical axonometric view from infinity.

The 'solar machine' and representational theory at the beginning of the 19th century

What prevents Accolti or Dubreuil from seeing what we today call an 'axonometric projection' in their shadows? As we have already seen with Cousin, there are domains of reality (the military, the human body) that claim their own representational domains, each with its own source of legitimacy. In other words, one does not conceive a universal scene under a single principle of representation. Still, it can happen – without this being shocking – that in the same scene, there coexist objects drawn with those procedures that are divergent but are proper to them. The stage of representation is not a coherent space but a place where 'objects appear'. For Accolti or Dubreuil, the 'iconographic shadow' and the proto-axonometric which it resembles would belong to domains of reality whose fields of representation did not need to coincide.[35]

For it to have been possible for Accolti or Dubreuil to have recognised axonometry in these shadows, two conditions would have needed to be met. The first is that there be a radical break in the implicit theory of representation they shared – that a new one appears in which there is a single basis for legitimacy, a 'system' that creates scenes in which all bodies, regardless of their origin, may be inserted within a coherent space. The second is that the source of this legitimacy be the sun, so that the flat shadow of an object can be identified with its axonometric projective image.

As we will see below, both circumstances will occur in the context of argumentation that laid the foundations of isometry in the early 18th century in England. Concerning the first condition, it is the first source of universal legitimacy that will appear – with William Farish – and will be 'visual', so that the idea of a system is split into the isometrical (when the spectator 'sees from' infinity) and perspectival (when the location of the observer in relation to the object is determined). (This would not yet favour recognising the similarity between a flat shadow and a body, since they seem to be different entities for 'the eye' that contemplates them.)

However, the 'spectator', as a basis of legitimisation, will soon be replaced in the work of some specific authors by a 'solar machine' that casts transparent shadows. Thus, the two systems (isometric and perspectival) correspond to shadows, either cast (respectively) by the sun or a lamp. The sun, which had played an ambivalent role as a non-human spectator in the ancient theory of representation, and which had made it possible to imagine virtual machines that solved 'local' drawing problems (the layout of the plan in architecture, the projections of regular bodies, etc.), becomes the potential universal foundation of representational procedures.

Isometry as a universal mode of representation: William Farish and the spectator at infinity

'Proto-axonometrics' had remained throughout the 17th and 18th centuries appropriate drawings for particular phenomena born of a sequence of graphical operations that 'coincidentally' produced a resemblance to the object from which they derived. At the end of the 18th century, the idea of legitimising them – as a view seen from a great distance, or even infinity – began to proliferate. But soon, this subjective spectator would be revealed to be problematic, not to say absurd. Proposals and intuitions in this sense (such as those of C.F. Milliet Dechales [1684] or George Fournier [1706]) would be eroded by the criticism and sceptical arguments of Johann Heinrich Lambert (1759) or Nicolas François Chevalier de Curiel (1777), which joined with those of Aguilones and Huret, which we have already mentioned.[36]

At last, the British scientist William Farish (1759–1837) found that locating the object in a particular position, and the spectator at infinity, seemed acceptable. In 1822, he published an article explaining the basis of his system.[37] In this, he avoids any geometrical or mathematical complexities, opting instead for a visual description – isometric perspective is the image of a cube seen by a spectator who has moved diagonally away from it indefinitely.[38] While Farish's system was initially intended for the representation of machines and mechanisms with wheels and gears, he would in fact break with the paradigm of representation centred on specific objects, realising that, although he had started its demonstration with a simple cube, his isometry described a space able to contain all things seen from infinity. Throughout his text a vocation emerges, an ambition for universality, which no 'proto-axonometric' text had hitherto postulated.

With enthusiasm, Farish points out the advantages of this position from which the viewer contemplates the whole scene of the world. In his text, he comments how it can be used to represent a building, a bridge, a cathedral, a college, a palace (including 'even the rooms and internal structure'[39]), a plan of a city, subterraneous objects, a ship, animals, a regular fortification (which was a sort of claim laid upon the preferred object of continental pre-axonometry), a mountainous country, or geological strata. All this implicitly requires moving from the idea that one is looking at a body (a cube) to the notion that one is projecting the space that contains it, measured in cubical units, a concept that would later be manifested very explicitly in a drawing by Edward Cresy (1792–1858), an architect and engineer of the next generation (Fig.12). It is interesting to note how the frame of the drawing is not a conventional rectangle, but a hexagon inscribed in a circle – a remnant of the 'cube' which gave rise to the system and which is now the natural boundary of a modular space in which a three-dimensional representation of the nave of Amiens cathedral can be accommodated.[40]

On the other hand, in this nascent phase of isometry, it is striking to find an echo of the imaginary transparent shadows Villalpando had conceived to forge a new definition of *ichnographia*. Farish even played with the possibility of drawing transparent isometries of objects, although he discarded it to avoid confusion (Fig.13).[41]

Thomas Sopwith's 'solar machine': the disappearance of the spectator

This 'model' of Farish's would initially be maintained by his epigones, such as Thomas Sopwith (1803–1879) who, in his *A Treatise on Isometrical Drawing* (1834), enthusiastically extended the domains of isometry and definitively broke down the walls that assigned the modes of representation to certain professions (and also gender, suggesting its teaching and use to 'ladies').[42]

Sopwith initially put forward the idea that isometry coincides with the perspective of a cube, whose position *vis-à-vis* the viewer allows the distortion on all sides to be the same when seen from infinity.[43] In demonstrating this, Sopwith is forced to follow a process that develops by successive approximations to a limit. He shows the reader views of a cube which, as it moves progressively away towards infinity along its diagonal, produces images that grow closer and closer to isometric projection (Fig.14). To be more convincing, he takes care to tabulate the measurements of these images. This is, for him, what gives isometry an advantage over other oblique projections – although they may have 'geometrical truth', they lack the 'visual truth' of isometry.[44]

But Sopwith then changes his strategy and develops a demonstration in which the spectator disappears, replaced by an 'objective' machine – empirical, alien to our subjectivity, and capable of generating the isometry of a cube 'immediately'. A wireframe model (real or virtual) can be created so that the sun draws 'transparent shadows' that match the isometry. To begin with, Sopwith describes how the ichnography of such an object can be obtained:

> The shadow of an object by the sun upon a plane perpendicular to its rays is the orthographical projection of the contour of the object, and if in solids comprised under plane surfaces, we construct, or supposed to be constructed, a frame or cage of wires, which shall form the same angles, and which shall have the same proportion to one another as the edges of the solid, the shadow of the frame by the sun upon a plane perpendicular to the rays of light, would be the *orthographical projection* of the linear edges of the solid, and exactly what ought to be drawn when the position of the object to the plane of projection is known.[45]

He goes on to explain how, from this 'cage' of wires, the sun can draw the 'Isometrical projection' of the solid it envelops (Fig.15):

> [If] the wire frame were similarly constructed to the edges of a solid comprised under rectangular planes, and the sun's rays parallel to the diagonal of a cube, which has its edges parallel to those of the wires, the shadow of this frame would be the isometrical projection of the linear edges of the solid.[46]

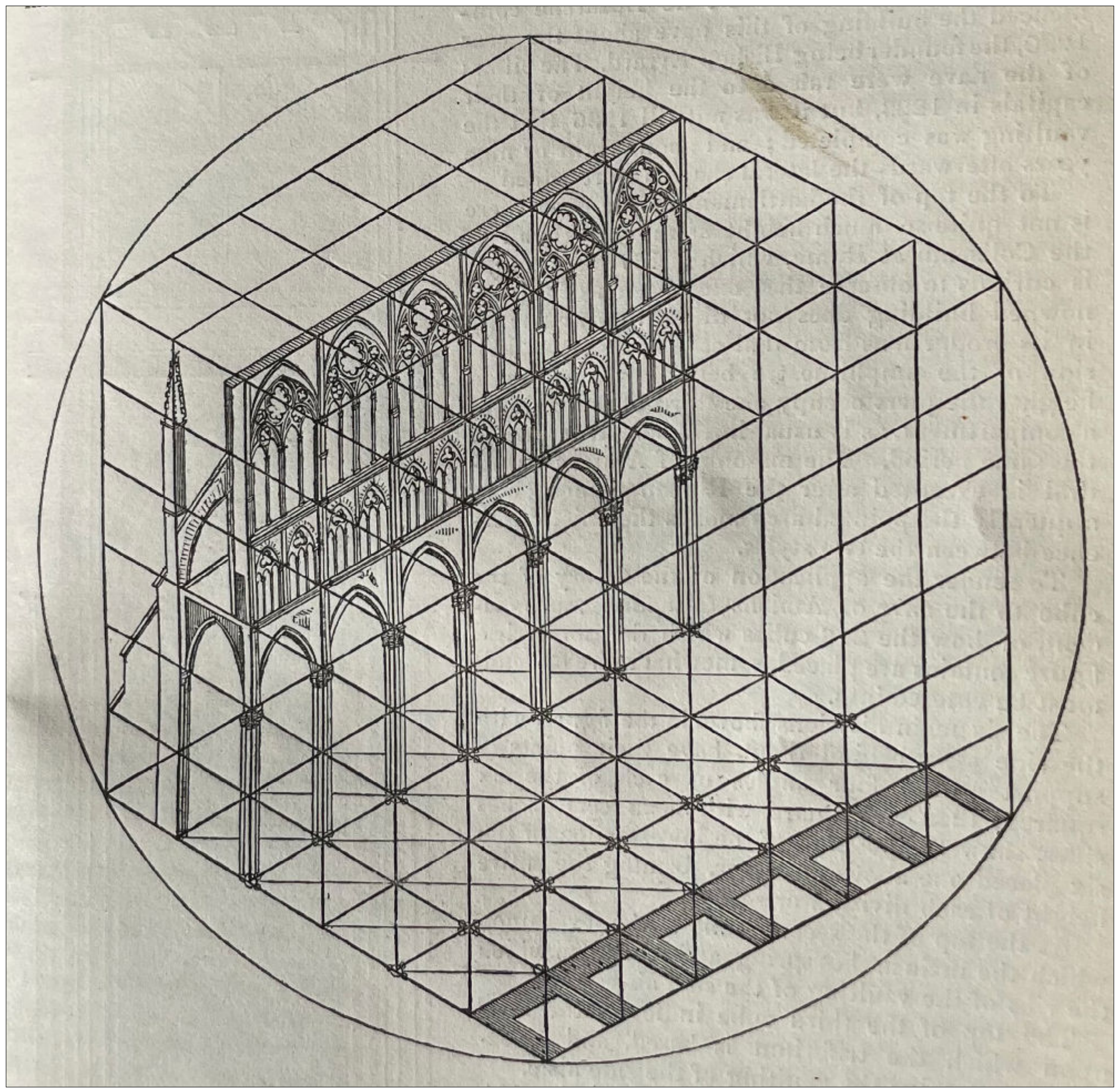
Moreover, Sopwith notes that this virtual machine shows that both isometric and perspective can now be understood as transparent shadows drawn, in the first case, by the sun and, in the second, by a candle:

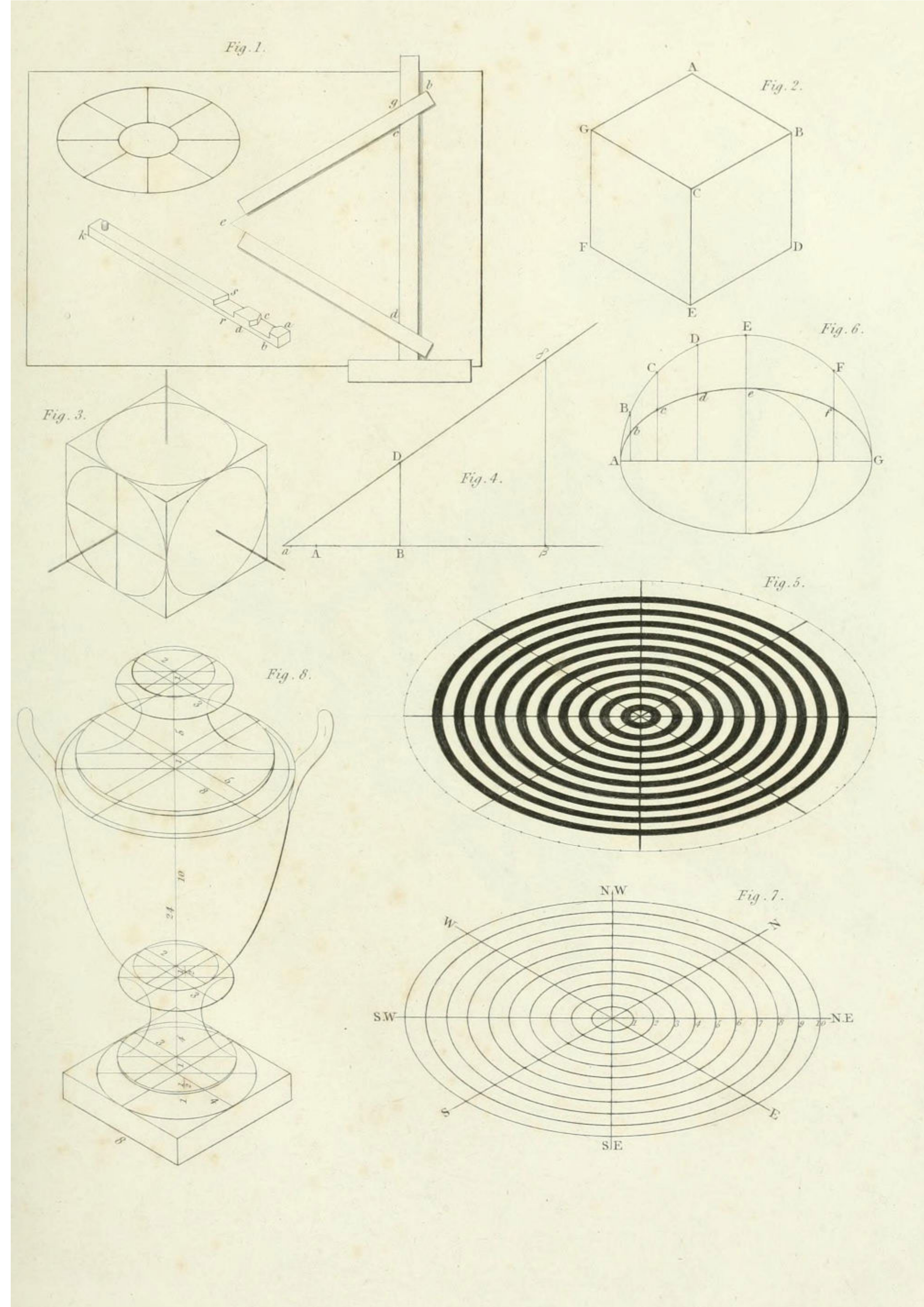
> [If] in a point at a limited distance from the object, the flame of a candle be supposed to be condensed, the shadow of the wire frame by this light, upon a plane behind it, would be the perspective representation of the linear edges of the solid; and if the light were in the diagonal produced of a cube similarly situated to the wire frame, and the plane of the picture perpendicular to this diagonal, we should have the isometrical perspective representation of the linear edges of the solid.[47]

This statement prefigures the creation of two representation systems sharing light as their fundament. This change of mental framework could finally allow us to read the transparent shadows of Dubreuil or Accolti as the image of an axonometric.[48] As Sopwith observed, the isometry of a cube produced a hexagon, a plane figure known since antiquity but only recently recognised as the image of a projection. Perhaps, in saying this, Sopwith had in mind some of the illustrations related to the problem of constructing the perspective of ideal bodies in the books of Cousin or Brook Taylor.[49]

Fig.12 Edward Cresy's drawing of Amiens Cathedral in a modulated isometrical space framed into a cube. From Edward Cresy; (engravings by R. Branston), *An Encyclopædia of Civil Engineering: Historical, Theoretical, and Practical*, new impression (London: Longmans, Green, Longman and Roberts, 1861), 1665. First edited as *Supplement to An Encyclopaedia of Civil Engineering, Historical, Theoretical, and Practical* (London: Longman, Brown, Green, and Longmans, 1856). Courtesy Biblioteca del Colegio de Ingenieros de Caminos, Canales y Puertos de Madrid.

Fig.13 Farish's devices and templates that facilitate isometric drawings in a plate including as an example a transparent version of a vase. From William Farish, 'On isometrical perspective', *Transactions of the Cambridge Philosophical Society*, I (1822), 1–20. Public domain, via Wikimedia Commons.





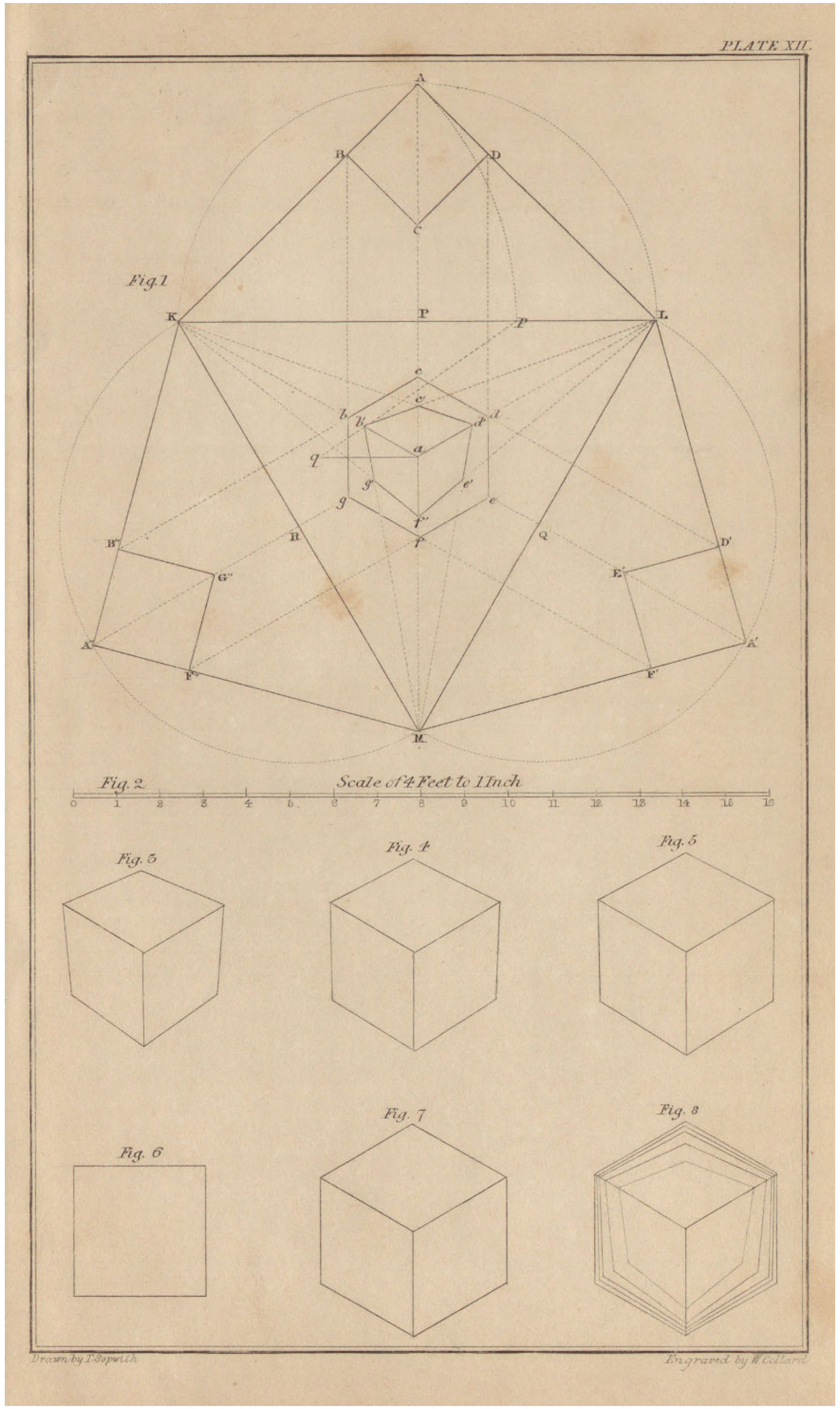
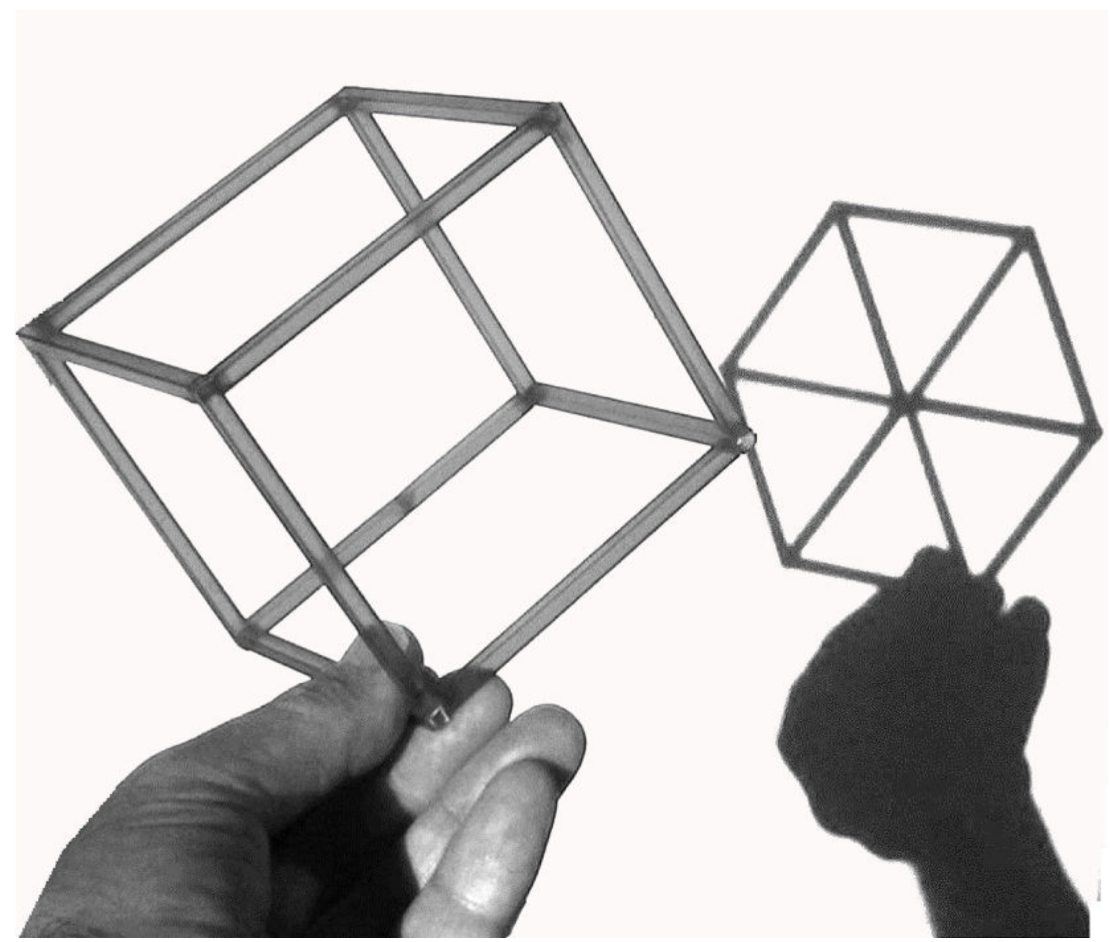


Fig.14 Diagrams demonstrating isometry as perspective from a viewpoint approaching infinity. From Thomas Sopwith, *A Treatise on Isometrical Drawing, and Applicable to Geological and Mining Plans, Picturesque Delineations of Ornamental Grounds, Perspective and Working Plans of Buildings and Machinery, and to General Purposes of Civil Engineering*, Second Edition (London: John Weale, 1838), plate XII. Public domain via e-rara.

Fig.15 A model constructed akin to Sopwith's 'wire cage' to test isometry as a shadow cast by the sun. Photo: author.



According to Sopwith, the same 'solar machine' that justifies isometrics prevents the rest of the 'proto-axonometries' from being legitimate. Sopwith finds that not all shadows are correct representations. Oblique shadows may bear little relation to the proportions of the object. For this reason, Sopwith prefers to consider them only as valuable drawings which are merely the result of graphic operations on paper. Thus, Sopwith calls what we know as cavalier axonometric 'verti-horizontal drawings', prescribing how they can be measured and constructed on oblique axes using scales and hand instruments designed for this purpose.[50]

Joseph Jopling on true and false projections: the morality of isometry

It is striking that only a year later, Joseph Jopling (1788–1867), in his version of Taylor's treatise on perspective (1835), explored the possibility of seeing some 'proto-axonometric' projections as shadows cast by the sun or the moon.[51] Jopling first defines the projection by beams of parallel lines orthogonal to a projection plane as 'direct radial projection'. He then points out that this relates to 'isometrical perspective', the plans or elevations of a building, as well as 'the shadows of any objects on any plane on which the sun or moon shines direct, as the rays of these (to all sense) are parallel to each other'.[52]

In the case that the plane of projection is tilted with respect to the rays, we would have an 'oblique radial projection': 'The shadows of any objects on any plane on which the sun or moon does not shine direct, are of this projection', Jopling points out. The diagrams accompanying his explanation show that what he has in mind are the shadows cast by objects resting on a plane that receive this beam of inclined rays (Figs 16a, b, c).[53] Then Jopling states a principle that would imply accepting that all 'proto-axonometries' are shadows: 'Oblique sections of any object … are the same as this projection'.[54] This is relevant because, given this formulation, one can now see, for example, the shadow drawn by Accolti in terms of a 'military perspective'. But he immediately concludes that such shadows are 'false', insofar as they misrepresent the dimensions of the object to which they refer:

> In fig.8 the sun is supposed to shine on each of the two faces of the cube … at an angle of 45°…. Thus … the greater the obliquity of the rays, the more the length of the shadow or projection exceeds the dimensions of the object in the other direction.[55]

As Jopling observes, slanting shadows can confound the relationship between objects, even making the shorter appear longer, and *vice versa*:

> If one object be long and another short, but in other respects the same, by a greater obliquity in the rays, the projection of the shorter may be made as long or in any degree longer than the other.[56]

From this mental experiment with shadows, Jopling concludes that to represent objects in oblique projection (as is the case with 'military' or 'cavalier' perspective) is to 'give them a false appearance'.[57] The only actual shadow is that of the isometric, that which 'seen from an infinite distance, or the sun or moon, appears the same, in whatever plane its shadow is cast'.[58] Ultimately, what prevents Jopling from accepting that there is an axonometric system with several variants is not only a geometrical argument but a somewhat moral one: although all 'proto-axonometric' projections can be acknowledged as shadows cast by the sun, only the isometric one has the force of truth.

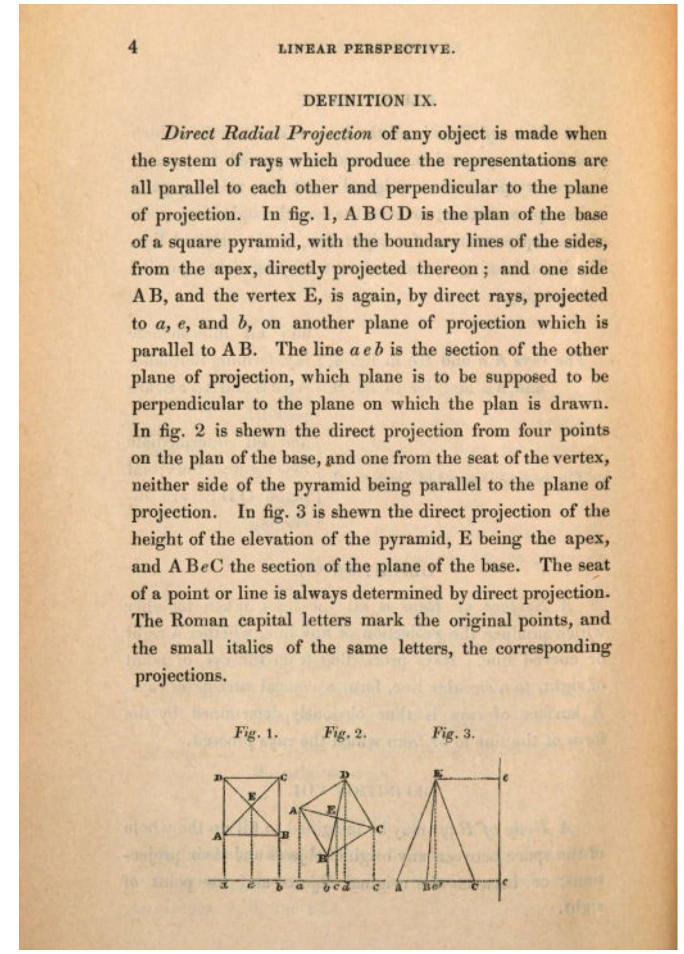
We end here, having seen how, in the early 19th century in England, there was a radical shift represented by the attempt to ground the isometric system and perspective on the same basis. At first, with Farish, this source of shared legitimacy was constituted by the presence of a spectator who, Sopwith thought, could be eventually replaced by 'light'. There are understandable reasons for this change. Sopwith would realise that the old *alter ego* of that spectator, the sun, could – in an exchange of roles – advantageously replace the viewer in the infinity of isometry. The transparent ichnographic shadows cast by the sun of an object rotated in a particular and concrete position in space were images equivalent to those contemplated by Farish's anthropomorphic spectator. Better still, the 'solar machine' constituted an excellent source of validation, allowing for an immediate, objective, empirical demonstration of isometry, which avoided all the paradoxes and difficulties posed by assuming a hypothetical viewer at infinity. The sun could even be disembodied – it was no longer necessary to see it as a 'non-human spectator'. It was simply a source of radiation that mechanically generated isometric shadows.

Once this approach had been elaborated, one was on the verge of recognising that the rest of the 'proto-axonometries' could also be seen as shadows of bodies cast from different angles by the sun's rays upon a horizontal plane.

Conclusion

We can summarise the main conclusions in three ideas: that between the end of the 16th and the beginning of the 19th centuries a 'virtual' solar drawing instrument was invented and developed; that this machine played a role that deserves to be considered in a comprehensive history of the concept of projection; and finally, that this machine, which was the fruit of a fusion of particular cultural

Figs 16a, 16b, 16c Joseph Jopling, projections as shadows, with (16c) the oblique as the false appearance of an object. From Joseph Jopling, *Dr Brook Taylor's Principles of Linear Perspective, a New Edition with Additions Intended to Facilitate the Study of this Much Extended Work, by Joseph Jopling* (London: M. Taylor, 1835), 4–6. Public domain, via HathiTrust.


4 LINEAR PERSPECTIVE.

DEFINITION IX.

Direct Radial Projection of any object is made when the system of rays which produce the representations are all parallel to each other and perpendicular to the plane of projection. In fig. 1, A B C D is the plan of the base of a square pyramid, with the boundary lines of the sides, from the apex, directly projected thereon; and one side A B, and the vertex E, is again, by direct rays, projected to *a*, *e*, and *b*, on another plane of projection which is parallel to A B. The line *a e b* is the section of the other plane of projection, which plane is to be supposed to be perpendicular to the plane on which the plan is drawn. In fig. 2 is shewn the direct projection from four points on the plan of the base, and one from the seat of the vertex, neither side of the pyramid being parallel to the plane of projection. In fig. 3 is shewn the direct projection of the height of the elevation of the pyramid, E being the apex, and A B*e*C the section of the plane of the base. The seat of a point or line is always determined by direct projection. The Roman capital letters mark the original points, and the small italics of the same letters, the corresponding projections.

Fig. 1. *Fig*. 2. *Fig*. 3.

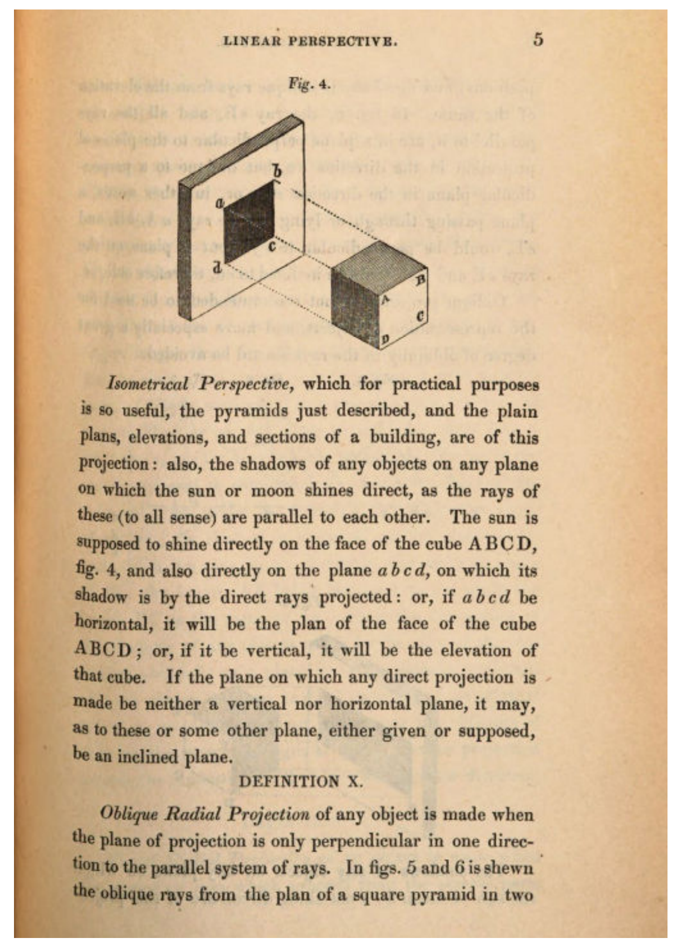
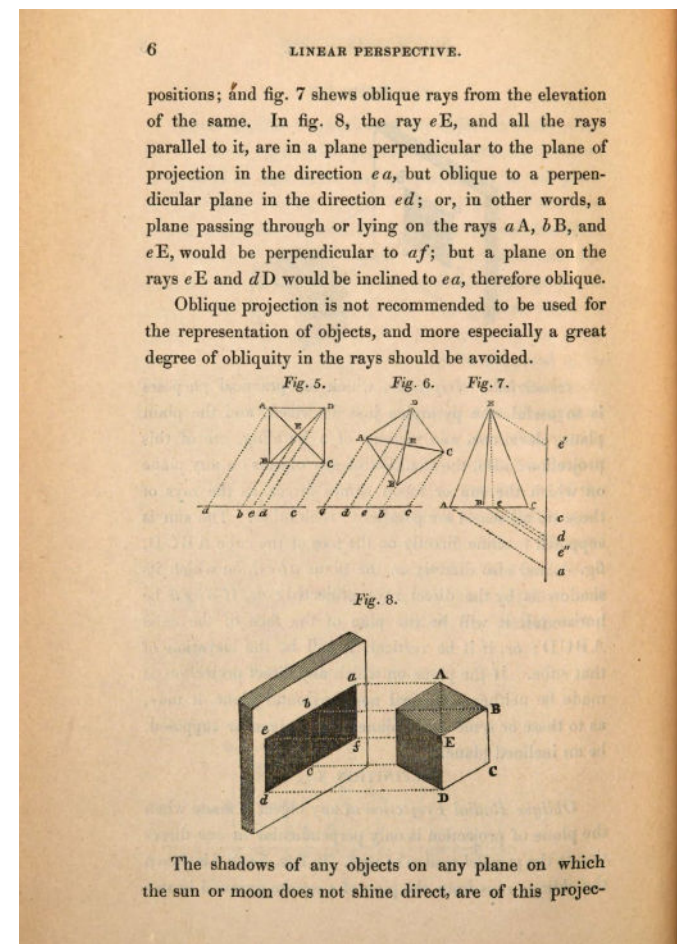

LINEAR PERSPECTIVE. 5

Fig. 4.

Isometrical Perspective, which for practical purposes is so useful, the pyramids just described, and the plain plans, elevations, and sections of a building, are of this projection: also, the shadows of any objects on any plane on which the sun or moon shines direct, as the rays of these (to all sense) are parallel to each other. The sun is supposed to shine directly on the face of the cube A B C D, fig. 4, and also directly on the plane *a b c d*, on which its shadow is by the direct rays projected: or, if *a b c d* be horizontal, it will be the plan of the face of the cube A B C D; or, if it be vertical, it will be the elevation of that cube. If the plane on which any direct projection is made be neither a vertical nor horizontal plane, it may, as to these or some other plane, either given or supposed, be an inclined plane.

DEFINITION X.

Oblique Radial Projection of any object is made when the plane of projection is only perpendicular in one direction to the parallel system of rays. In figs. 5 and 6 is shewn the oblique rays from the plan of a square pyramid in two


6 LINEAR PERSPECTIVE.

positions; and fig. 7 shews oblique rays from the elevation of the same. In fig. 8, the ray *e*E, and all the rays parallel to it, are in a plane perpendicular to the plane of projection in the direction *e a*, but oblique to a perpendicular plane in the direction *ed*; or, in other words, a plane passing through or lying on the rays *a*A, *b*B, and *e*E, would be perpendicular to *af*; but a plane on the rays *e*E and *d*D would be inclined to *ea*, therefore oblique.

Oblique projection is not recommended to be used for the representation of objects, and more especially a great degree of obliquity in the rays should be avoided.

Fig. 5. *Fig*. 6. *Fig*. 7.

Fig. 8.

The shadows of any objects on any plane on which the sun or moon does not shine direct, are of this projecpresuppositions, calls into question a purely logical account of the birth and evolution of the concept of projection.

This machine was based on the idea of placing a transparent version of the body to be drawn under the sun and using its diaphanous shadow as an objective representation of it. Its source (the sun), the emission (luminous radiation), and the projected figure (a transparent shadow) appealed to earthly empirical experience, allowing an approach to the abstract concept of parallel projection. This 'solar machine' facilitated painterly explorations of the foreshortening of the human body (as we have seen in the case of Cousin), enabled mathematicians and scholars of perspective to define the representation of geometric bodies floating in space on a plane (Dubreuil), and supplied an intermediate step for those (such as Accolti) who sought to determine cast shadows in perspective.

However, during the period the 'solar machine' conquered only limited territories, as growing doubts about its status arose. Was it a perspective, and if so, who saw it? A divine sun, or a frustrated sun-eye that does not see the shadows it draws, as a puzzled Accolti noted? In addition, it evolved in a visual culture in which bodies (platonic solids, buildings, or fortresses) could claim their own form of representation. This was a significant impediment to the development of a universal concept of parallel projection.

With Sopwith, at the beginning of the 19th century, a radical change took place. Farish had postulated isometry as a system capable of generating a coherent space in which any object can be inscribed and to which the drawing of any profession can be attached. But his definition of isometry as the perspective of an eye approaching infinity was somehow unsatisfactory. Sopwith realised that the 'solar machine' offered a better solution. It was only necessary to renounce the idea that any distanced spectator was needed. Isometry was, substantially, a palpable sensible empirical shadow cast by the sun of a box, a kind of spatial module of isometric space extending in all directions, placed in a particular position relative to the plane of projection. The 'solar machine' offered objective, empirical, irrefutable proof that isometry was possible.

This decision might be viewed as transforming the 'solar machine' into a universal virtual drawing instrument applicable to any object. It heralded a new stage in which all parallel projections (including axonometry and plan) could be conceived in the same way. The diaphanous module could have adopted other positions under the light and generated axonometric shadows corresponding to cavalier or military perspective but the British promoters of isometry, Sopwith and Jopling, were reluctant to take this step.

Throughout this study, we have verified how the concept of parallel projection, which today we appreciate as a logical and rational construction, took shape in relation to a particular magma of ideas, in which myth and thaumaturgical and symbolic thought had a place. The machine's components – the sun and the transparent body – were based on specific cultural premises. It fused a myth inherited from our classical culture with the subtle Renaissance concept of the *corpo transparente*, which arose from the enigmatic Neoplatonic doctrines of Luca Pacioli and was recommended by Sebastiano Serlio as a figure expressing the mental penetration of the hidden features of any object. The sun, often understood symbolically as an eye, appeared, in some cases – like that of Villalpando, with which we began – wrapped in the peculiar religious and philosophical atmosphere of the Counter-Reformation, which imbued light with a transcendental significance.

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1 The concept applied here is an extension of the common idea of a machine as an apparatus or device composed of elements that takes advantage of an external energy source in order to produce a particular effect or movement. In the meaning I use here, it is a device that 'passively' (in that it does not require much articulation through internal components) uses a natural force or agent, such as gravity or light rays, to produce on a plane a two-dimensional picture that corresponds to a three-dimensional object. In this sense, a plumb line, a wire cube, and a camera obscura would all be machines. At the same time, machines of this sort do not need to be material or physical objects – they can be speculative theoretical models. The desired goal is to generate drawings, or graphic substitutes, without human intervention, thereby eliminating any subjectivity. In this way, their images acquire the status of the true and positive, as opposed to those which are contingent on the perception and artistic and manual skills of a human agent and are therefore fallible.

2 Vitruvius, *On Architecture*, I,. ii, trans. Richard Schofield and Robert Tavernor (London: Penguin Classics, 2009).

3 I will use this neologism to distinguish those drawings, often not understood as projections, from those which follow the projective rule. The term axonometry was coined by M.H. Meyer, *Lehrbuch der axonometrischen Projektionslehre* (Leipzig, 1863).

4 Pliny the Elder, *Natural History*, vol.9, 33–35, trans. H. Rackham, Loeb Classical Library (Cambridge, MA: Harvard University Press, 1984) and *The Instituto Oratoria of Quintilian*, X, ii, 7, trans. H. Edgeworth Butler, Loeb Classical Library (Cambridge, MA: Harvard University Press; London, William Heinemann Ltd., 1922).

5 Regarding the use of shadow in the empirical investigation of perspective, see George Bauer, 'Experimental shadow casting and the early history of perspective', *The Art Bulletin*, vol.69, no.2 (June 1987), 211–19.

6 Victor I. Stoichita, *A Short History of the Shadow* (London: Reaktion Books, 1997). Of particular interest here are chapters one and two, 11–88.

7 On this issue, see George L. Hersey, *Pythagorean Palaces: Magic and Architecture in the Italian Renaissance* (Ithaca: Cornell University Press, 1976).

8 Author's translation from Sebastiano Serlio, *Il Primo (-secondo) Libro d'Architettura* (In Vinetia [Venice]: per Cornelio de Nicolini da Sabbio: a instantia de Marchio Sessa, 1551), 35–36.

9 In a manuscript (c.1560) by the Spanish architect Hernán Ruiz el Joven we find a clear early example of the influence of Serlio's idea on architectural drawing: see the pavilion drawn in both versions (opaque and transparent) in Pedro Navascués Palacio, *El Libro de Arquitectura de Hernán Ruiz el Joven* (Madrid: ETSAM, 1974) lam. LI, y LII folio 52, y 53.

10 Bernardino Amico da Gallipoli, *Trattato delle Piante e immagini de I Sacri Edifizi di Terra Santa Disegnate in Gierusalemme secondo le regole della Prospettiva, & vera misura della lor Grandeza* (Rome, 1609), plate 24. The explanatory comment is found in the second edition (Firenze: Pietro Cecconcelli, 1620, 54, 'Corpo transparente della passata Chiesa', plate 43). How Amico 'crystallises' and gives form to the architectural void does not seem to have precedents in other graphic cultures.

11 He probably had contact with figures of the stature of Christoph Clavius at the Roman College. It should be no surprise that later Isaac Newton referred to Villalpando when tackling his own reconstruction of Solomon's temple.

12 Juan Bautista Villalpando and Jerónimo del Prado, *In Ezechielen Explanationes et Apparatus Urbis ac Templi Hierosolomitany*, 3 vols (Rome: ex typographia Aloysij Zannetti, 1595–1606). Here I have referred to the second part of vol.2, entitled *De postrema Ezechielis Prophetae visione*. This section deals with architectural issues and has been translated into Spanish as a stand-alone book: Juan Bautista Villalpando, *El Tratado de la Arquitectura Perfecta en La última vision del profeta Ezequiel*, ed. José Corral Jam, trans. Fray Luciano Rubio O.S.A. Editor (Madrid: Colegio Oficial de Arquitectos de Madrid, Patrimonio Nacional, 1990). I focus on part two,'Explanations of the graphic descriptions of the temple', 135–90, with special attention to chap. 2, 169–71.

13 Villalpando, *op. cit.*, 169.

14 *Ibid.*, 170.

15 *Ibid.* The translation into Spanish from the original Latin uses the phrase 'edificio modico', which I am rendering as 'model'. Villalpando's use of the phrase 'seen by an eye equal to the same building' signifies a hypothetical ocular position capable of comprehending the parallel rays of orthographic projection.

16 *Ibid.*

17 *Ibid.*, 171.

18 *Ibid.*

19 Juan Caramuel, 'Tratado VI en que se enseña la architectura obliqua, Artículo III, De la ichnographia o sciographia', vol. II, *Architectura Civil, Recta, y Obliqua, Considerada y Dibuxada en el Templo de Ierusalem* (Vigevano: Camillo Corrado, 1678), 4–5.

20 *Ibid.*, also in Tratado VI: 'Artículo VI, De qué figura han de ser las bases y las columnas que se pusieren en un edificio circular', 9 and plate XLV.

21 Indicative of this is the progressive substitution during Villalpando's lifetime of the traditional pavilion monstrance that houses the sacred host by the 'sun monstrance'. For an overview of the symbolic role of the sun in Jesuit thought see Kevin Duffy, *Christian Solar Symbolism and Jesus the Sun of Justice* (London: Bloomsbury, 2022), 61–63.

22 Pérez-Gómez argues that Caramuel and Villalpando's concept of projection, much permeated by their religious ideas, differs from later and more abstract developments. See the section 'Sciographia and projected shadows', in Alberto Pérez-Gómez and Louise Pelletier, *Architectural Representation and the Perspective Hinge* (Cambridge, MA, and London: MIT Press, 1997), in particular 119–24. Quote from p.123.

23 Robin Evans, *The Projective Cast. Architecture and Its Three Geometries*, (Cambridge, MA, and London: MIT Press, 1995), 1–47.

24 Caramuel, *op. cit.*, vol. II, Tratado VII, 'De algunas artes o ciencias que acompañan y adornan a la arquitectura Artículo I, De la Pintura', 41.

25 *Livre de pourtraicture de Maistre Jean Cousin, peintre et géométrien très-excellent* (Paris: Le Clèrc, 1595). Reissued as Jean Cousin, *La Vraye science de la pourtraicture* (Paris: chez Guillaume Le Bé, 1647). The book was perhaps based on the work of Cousin's father and has been regularly reprinted (24 editions by 1909). For a review of the work of father and son see Cécile Scailliérez and Helène Billat, *Jean Cousin père et fils. Une famile de peintres au XVIe siècle* (Paris: Musée du Louvre, 2013).

26 In his *Traité complet de peinture*, vol.6 (Paris: J.-F. Delion, 1829–51), 197, Jacques Nicolas Paillot de Montabert defends Cousin's approach, saying that 'there is nothing against this new theory' and pointing out that it has been suggested and tried by 'many geometrical painters' such as Albrecht Dürer, Paul Lomazzo, Daniele Barbaro, Bernardo-Zénale, and Vincenzo Foppa.

27 Grégoire Huret, *Optique de portraiture et peinture, d'optique en deux parties* (Paris: Chez l'auteur, 1670), 84. This attack on Cousin also targets Daniele Barbaro 'for wanting to extend Albrecht Dürer's thinking beyond what he did'. This aversion to placing the eye at infinity had already arisen in the second half of the 16th century. Guidobaldo Bourbon Del Monte, in book II of his *Planisphaeriorum Universalium Theorica* (Pesaro: apud Hyeronimum Concordiam 1579), 58, reacts against the reasoning that justified the construction of the astrolabe and the universal planisphere because 'placing the Eye at an infinite distance meant putting it in no place', a concept that 'is abhorrent to perspective itself'.

28 Pietro Accolti, *Lo inganno de gl'occhi, prospettiva pratica* (Florence: Appresso Pietro Cecconcelli, 1625). For a historical overview of the problem addressed by Accolti see Thomas DaCosta Kaufmann, 'The perspective of shadows: the history of the theory of shadow projection', *Journal of the Warburg and Courtauld Institutes*, vol.38 (1975), 258–87. Reprinted in Thomas DaCosta Kaufmann, *The Mastery of Nature. Aspects of Art, Science and Humanism in the Renaissance* (Princeton, N.J.: Princeton U.P., 1993), 49–78.

29 'Onde si come specolando intendiamo il Sole non vedere giamai alcuna ombra degl'opachi, & superficie, ch'egli rimiri,& illustri, cosi tutte quelle, che vengono in sua veduta, intendiamo restar lummegiate, & per il contrario tutte l'altre a lui ascose restar ombreggiate , & prive di suo splendore.' Accolti, *op. cit.*, 139.

30 Filippo Camerota, 'The eye of the sun: Galileo and Pietro Accolti on orthographic projection', in *Perspective, Projections & Design. Technologies of Architectural Representation*, eds Mario Carpo and Frédérique Lemerle (London and New York: Routledge, 2008), 115–25. Camerota pointed out that the considerations made by Accolti reflected Galileo's arguments in his *Siderius Nuncius* (Venice, 1610) supporting his interpretation of the orographic nature of the moon.

31 Leonardo da Vinci had articulated it thus: 'If the sun is in the east and you look towards the west, you will see all things illuminated, for you see what the sun sees; if you look towards the noon or the north, you will see all bodies enveloped by light and shadow, for you see what the sun does not see.' B.N. 2038, 18b *Codex Urbinas Latinus*, Biblioteca Apostolica Vaticana, here translated from Ángel González García, *Leonardo da Vinci, Tratado de Pintura* (Torrejón de Ardoz, Madrid: Ediciones Akal, 1986), 181. On the other hand, the image of the human eye as sun was also circulating, eventually facilitating an exchange of roles in the theory of 'solar machines'. See, for example, François Niceron, *L'Optique et la catoptrique du reverend Père Mersenne minime* (Paris: Veuve F. Anglois, 1651), 60: 'The eye is like the Sun of man'.

32 Jean Dubreuil, *La Perspective pratique, necessaire a tous peintres, graveurs, sculpteurs, architectes, orphevres, brodeurs, tapissiers, & autres qui se meslent de desseigner* (Paris: Antoine Dezallier, 1679), 'Avis au lecteur', unnumbered page.

33 *Ibid*., 99.

34 *Ibid*., 100.

35 For an example of the co-existence of different modes of representation see Alain Manesson Mallet, *Les Travaux de Mars* (Paris: Denys Thierry, 1685), Fig. LXVI, 161, in which a curved perspective scene progressively becomes a map (cf. Jacques Callot, *The Siege of Breda*, 1626–28, in the Museo del Prado), effortlessly accommodating a 'proto-axonometric view' of a fortified citadel.

36 I will deal with this issue in greater depth in a subsequent publication, but collect here a few quotations to give an idea of its controversial development. George Fournier, in his *Traité des fortifications ou architecture militaire* (Paris: Jean Henault, 1648), saw 'military perspective' as a zenithal view, 'the eye being infinitely raised over the centre of the square' (65–66). For his part, Milliet Dechales, in *L'Art de fortifier, de defender et d'attaquer les places* (Paris: Estienne Michallet, 1684), took it as true perspective seen from 'a very distant point of view' (403). But those assumptions led to embarrassing conclusions that would impede acceptance. In *La Perspective affranchie de l'embarras du plan géometral* (Zurich: Heideggueret Comp, 1759), Johan Heinrich Lambert conceded that 'in orthographical projection, use is made of a point of view infinitely distant' . However, this perspective lacks a positive basis – it is only valid when our eyes observe tiny objects such as 'insects, or small instruments' and when it is used for 'larger machines, whole cities or fortresses cannot be expected to look natural' (148–66). In his article 'Perspective cavalière et militaire', in *Supplément à l'Éncyclopédie ou Dictionnaire raisonée des sciences, des arts*, vol. IV (Amsterdam: chez Mme Rey, 1777), 304–305, Nicolas François Chevalier de Curel arrives at a disturbing conclusion: that the view from such a point in the infinity will have no frame, ' [f]or if the eye is infinitely distant, the sphere of vision will be infinite; and if it travels successively through all the visual rays, there is nothing to prevent this supposition from being extended as far as one wishes'.

37 William Farish, 'On isometrical perspective', *Transactions of the Cambridge Philosophical Society*, I (1822), 1–20.

38 *Ibid*., 2–3. Here he suggests starting a perspective from a near point where the eye could be placed 'on the line formed by producing the diagonal of the cube'. But considering it not good enough for picturing machines, he proposes that 'the distance of the eye, and consequently that of the paper, be indefinitely increased, so that the size of the object may be negligible in relation to it'. Farish states that this way he would eventually reach a point where 'all lines drawn from any point of the object to the eye can be considered as perpendicular to the image, which becomes, therefore, a sort of orthographic projection' (5).

39 *Ibid*., 12–13.

40 Edward Cresy showed how 'Within an isometrical cube may be placed the entire nave of Amiens' in his *Supplement to An Encyclopaedia of Civil Engineering, Historical, Theoretical, and Practical* (London: Longman, Brown, Green, and Longmans, 1856), reproduced later in *An Encyclopædia of Civil Engineering: Historical, Theoretical, and Practical*, New impression (Longmans, Green and Co., 1872), 1665.

41 Farish, *op. cit*., 13: 'But in thus exhibiting buildings as transparent, and their interior laid open, there is a danger of being confused by a multiplicity of lines, which is a difficulty in a building containing many rooms, that would need some address to get over. It is better adapted to exhibit the inside of a single room, of a Cathedral, for instance, the aisles, and transepts of which would not cause any great perplexity.'

42 Thomas Sopwith, *A Treatise on Isometrical Drawing, and Applicable to Geological and Mining Plans, Picturesque Delineations of Ornamental Grounds, Perspective and Working Plans of Buildings and Machinery, and to General Purposes of Civil Engineering* (London: John Weale, Taylor's Architectural Library, 1834), 194.

43 *Ibid*., 69–70.

44 *Ibid*., plate XII.

45 *Ibid*., 73.

46 *Ibid*., 73–74.

47 *Ibid*., 76–77.

48 Sopwith lets us see how a new representation theory is emerging, which calls for the avoidance of any ambiguity between these two domains: 'The word projection is used by writers in a general sense, either for the perspective or for the orthographical representation of an object. The celebrated Brook Taylor, in his new principles of linear perspective, uses the word projection, and the words perspective representation, as synonymous, viz. the former in the sense of the latter. Other writers on perspective, who have not treated of orthographical projection, have used the same expressions indifferently for the perspective figure of the object. To avoid this ambiguity, the word projection is here used to signify orthographical projection, and perspective representation for the figure of the object or objects in perspective'. *Ibid*., 78.

49 Sopwith claims that 'the origin of isometrical projection is at least co-existent with the inscription of a hexagon in a circle, for if all the opposite angles of a hexagon be joined by straight lines, a perfect isometrical representation of a cube is thereby produced; but while these few obvious geometrical principles claim so great antiquity, their application to projection is altogether modern'. *Ibid*., 73. This 'pre-isometric' image appears in several treatises when dealing with the perspective of regular bodies, including Dubreuil, as we have seen.

50 *Ibid*., 139–42 and plate XVII.

51 Joseph Jopling, *Dr Brook Taylor's Principles of Linear Perspective, a New Edition with Additions Intended to Facilitate the Study of this Much Extended work, by Joseph Jopling* (London: M. Taylor, 1835). Jopling had recently published a book aiming to facilitate the dissemination of isometry, *The Practice of Isometrical Perspective* (London: Taylor, 1835).

52 *Ibid*., 4–5, figs 1–4.

53 *Ibid*., 5–6, figs 5–8.

54 *Ibid*., 7.

55 *Ibid*.

56 *Ibid*.

57 *Ibid*.

58 *Ibid*.